MTH 1125 Test #1 - (12 pm class) - Solutions

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Pat Rossi Name ______________________

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-7} =$

Step #1 Try Plugging In:

$\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-7} = \frac{2(2)^2+(2)+4}{(2)^2+5(2)-7} = \frac{14}{7} = 2$

i.e., $\lim_{x\to 2} \frac{2x^2+x+4}{x^2+5x-7} = 2$

2. Compute: $\lim_{x\to 6} \frac{x^2-8x+12}{x^2-4x-12} =$

$\lim_{x\to 6} \frac{x^2-8x+12}{x^2-4x-12} = \frac{(6)^2-8(6)+12}{(6)^2-4(6)-12} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 6} \frac{x^2-8x+12}{x^2-4x-12} = \lim_{x\to 6} \frac{(x-2)(x-6)}{(x+2)(x-6)} = \lim_{x\to 6} \frac{(x-2)}{(x+2)} = \frac{(6)-2}{(6)+2} = \frac{4}{8} = \frac{1}{2}$

i.e., $\lim_{x\to 6} \frac{x^2-8x+12}{x^2-4x-12} = \frac{1}{2}$

3. Compute: $\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-x-6} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-x-6} = \frac{(3)^2+4(3)-9}{(3)^2-(3)-6} = \frac{12}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 3^-} \frac{x^2+4x-9}{x^2-x-6} = \lim_{x\to\ 3^-} \frac{x^2+4x-9}{(x+2)(x-3)} = \frac{12}{(5)(-\varepsilon)} = \frac{\left(\frac{12}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x - 3 < 0 \end{array}$$

$$\lim_{x\to\ 3^+} \frac{x^2+4x-9}{x^2-x-6} = \lim_{x\to\ 3^+} \frac{x^2+4x-9}{(x+2)(x-3)} = \frac{12}{(5)(+\varepsilon)} = \frac{\left(\frac{12}{5}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x - 3 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 3} \frac{x^2+4x-9}{x^2-x-6}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{x^4+6x^3-5}{3x^6+7x^2-8x} =$

$$\lim_{x\to\ -\infty} \frac{x^4+6x^3-5}{3x^6+7x^2-8x} = \lim_{x\to\ -\infty} \frac{x^4}{3x^6} = \lim_{x\to\ -\infty} \frac{1}{3x^2} = 0$$

i.e., $\lim_{x\to\ -\infty} \frac{x^4+6x^3-5}{3x^6+7x^2-8x} = 0$

5. $f(x) = \frac{x^2+x-20}{x^2-9}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 9 = 0$

$\Rightarrow (x+3)(x-3) = 0$

$\Rightarrow x = -3$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -3^-} \frac{x^2+x-20}{x^2-9} = \lim_{x\to\ -3^-} \frac{x^2+x-20}{(x+3)(x-3)} = \frac{-14}{(-\varepsilon)(-6)} = \frac{-14}{(\varepsilon)(6)} = \frac{\left(-\frac{14}{6}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{ll} & x \to -3^- \\ \Rightarrow & x < -3 \\ \Rightarrow & x+3 < 0 \end{array}$$

$$\lim_{x\to\ -3^+} \frac{x^2+x-20}{x^2-9} = \lim_{x\to\ -3^+} \frac{x^2+x-20}{(x+3)(x-3)} = \frac{-14}{(+\varepsilon)(-6)} = \frac{14}{(+\varepsilon)(6)} = \frac{\left(\frac{14}{6}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{ll} & x \to -3^+ \\ \Rightarrow & x > -3 \\ \Rightarrow & x+3 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -3$ *is a vertical asymptote.*

$$\lim_{x\to\ 3^-} \frac{x^2+x-20}{x^2-9} = \lim_{x\to\ 3^-} \frac{x^2+x-20}{(x+3)(x-3)} = \frac{-8}{(6)(-\varepsilon)} = \frac{8}{(6)(\varepsilon)} = \frac{\left(\frac{8}{6}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \end{array}$$

$$\lim_{x\to 3^+} \frac{x^2+x-20}{x^2-9} = \lim_{x\to 3^+} \frac{x^2+x-20}{(x+3)(x-3)} = \frac{-8}{(6)(+\varepsilon)} = \frac{\left(-\frac{8}{6}\right)}{(\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \end{array}$$

Since the one-sided limits are **infinite,** $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+x-20}{x^2-9} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

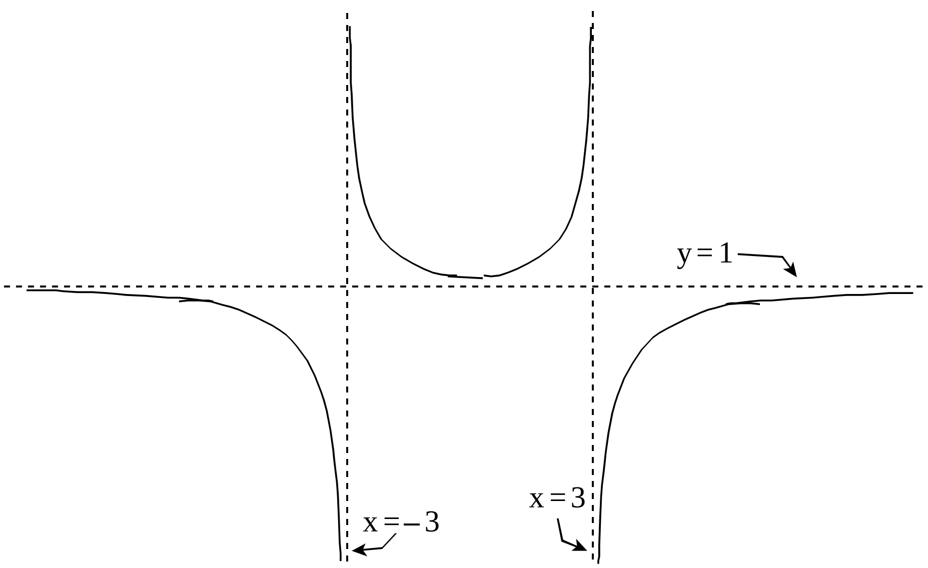
$\lim_{x\to+\infty} \frac{x^2+x-20}{x^2-9} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-3^-} \frac{x^2+x-20}{x^2-9} = -\infty$	
$\lim_{x\to-3^+} \frac{x^2+x-20}{x^2-9} = +\infty$	$\lim_{x\to-\infty} \frac{x^2+x-20}{x^2-9} = 1$
$\lim_{x\to3^-} \frac{x^2+x-20}{x^2-9} = +\infty$	$\lim_{x\to+\infty} \frac{x^2+x-20}{x^2-9} = 1$
$\lim_{x\to3^+} \frac{x^2+x-20}{x^2-9} = -\infty$	

Graph $f(x) = \frac{x^2+x-20}{x^2-9}$



6. Compute: $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} =$

Step #1 Try Plugging in:

$$\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \frac{\sqrt{(9)-5}-2}{(9)-9} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} \cdot \frac{\sqrt{x-5}+2}{\sqrt{x-5}+2} = \lim_{x\to 9} \frac{\left(\sqrt{x-5}\right)^2-(2)^2}{(x-9)\left[\sqrt{x-5}+2\right]}$$

$$= \lim_{x\to 9} \frac{(x-5)-4}{(x-9)\left[\sqrt{x-5}+2\right]} = \lim_{x\to 9} \frac{(x-9)}{(x-9)\left[\sqrt{x-5}+2\right]} = \lim_{x\to 9} \frac{1}{\left[\sqrt{x-5}+2\right]}$$

$$= \frac{1}{\left[\sqrt{(9)-5}+2\right]} = \frac{1}{[2+2]} = \frac{1}{4}$$

i.e., $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \frac{1}{4}$

7.

$x =$	$f(x) =$
-2.5	3.6
-2.1	30.8
-2.01	318.9
-2.001	$3,241.9$
-2.0001	$35,342.2$

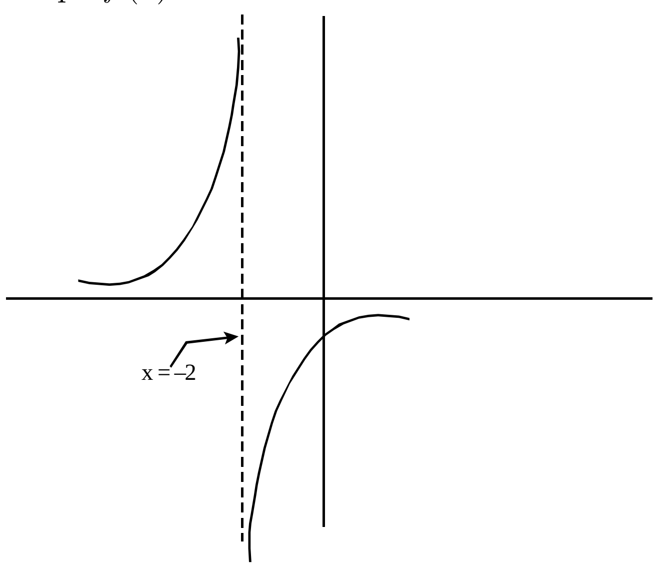
$x =$	$f(x) =$
-1.5	-3.6
-1.9	-30.8
-1.99	-318.9
-1.999	$-3,241.9$
-1.9999	$-35,342.2$

Based on the information in the table above, do the following:

(a) $\lim_{x \to -2^-} f(x) = +\infty$

(b) $\lim_{x \to -2^+} f(x) = -\infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 2$.

$$f(x) = \begin{cases} 2x + 2 & \text{for } x < 2 \\ 6 & \text{for } x = 2 \\ 5x - 4 & \text{for } x > 2 \end{cases}$$

If $f(x)$ is continuous at the point $x = 2$, then $\lim_{x \to 2} f(x) = f(2)$.

To see if this is true, we'll compute $\lim_{x \to 2} f(x)$.

Since the definition of $f(x)$ changes at $x = 2$, we must compute the one-sided limits in order to determine whether the limit exists.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 2) = 2(2) + 2 = 6$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (5x - 4) = 5(2) - 4 = 6$

Since the one-sided limits are EQUAL, $\lim_{x \to 2} f(x)$ exists and is equal to the common value of the one-sided limits.

i.e., $\lim_{x \to 2} f(x) = 6$

Furthermore, $f(2) = 6$

Hence: $\lim_{x \to 2} f(x) = f(2)$

Therefore, $f(x)$ IS continuous at $x = 2$