MTH 1125 Test #1

SPRING 2013

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2-2}{x+4} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3} \frac{x^2-2}{x+4} = \frac{(3)^2-2}{(3)+4} = \frac{7}{7} = 1$$

$$\boxed{\text{i.e., } \lim_{x\to 3} \frac{x^2-2}{x+4} = 1}$$

2. Compute: $\lim_{x\to 3} \frac{x^2-4x+3}{x^2+x-12} =$

Step #1 Try Plugging in:

$$\lim_{x\to 3} \frac{x^2-4x+3}{x^2+x-12} = \frac{(3)^2-4(3)+3}{(3)^2+(3)-12} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3} \frac{x^2-4x+3}{x^2+x-12} = \lim_{x\to 3} \frac{(x-1)(x-3)}{(x+4)(x-3)} = \lim_{x\to 3} \frac{(x-1)}{(x+4)} = \frac{(3)-1}{(3)+4} = \frac{2}{7}$$

$$\boxed{\text{i.e., } \lim_{x\to 3} \frac{x^2-4x+3}{x^2+x-12} = \frac{2}{7}}$$

3. Compute: $\lim_{x\to 3} \frac{x+2}{x^2-x-6} =$

Step #1 Try Plugging in:

$$\lim_{x\to 3} \frac{x+2}{x^2-x-6} = \frac{(3)+2}{(3)^2-(3)-6} = \frac{5}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good!. "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to 3^-}\frac{x+2}{x^2-x-6}=\lim_{x\to 3^-}\frac{x+2}{(x-3)(x+2)}=\frac{5}{(-\varepsilon)(5)}=\frac{1}{(-\varepsilon)}=-\infty$$

$$\begin{array}{|ll|}\hline & x\to 3^- \\ \Rightarrow & x<3 \\ \Rightarrow & x-3<0 \\ \hline\end{array}$$

$$\lim_{x\to 3^+}\frac{x+2}{x^2-x-6}=\lim_{x\to 3^+}\frac{x+2}{(x-3)(x+2)}=\frac{5}{(+\varepsilon)(5)}=\frac{1}{+\varepsilon}=+\infty$$

$$\begin{array}{|ll|}\hline & x\to 3^+ \\ \Rightarrow & x>3 \\ \Rightarrow & x-3>0 \\ \hline\end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to 3}\frac{x+2}{x^2-x-6}$ **Does Not Exist!**

4. $f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{for } x<3 \\ 6 & \text{for } x=3 \\ 2x-1 & \text{for } x>3\end{cases}$ Determine whether or not $f(x)$ is continuous at the point $x=3$. (Justify your answer.)

If $f(x)$ is continuous at the point $x=3$, then $\lim_{x\to 3}f(x)=f(3)$.

To see if this is true, we'll compute $\lim_{x\to 3}f(x)$.

Since the definiton of $f(x)$ changes at $x=3$, we must compute the one-sided limits in order to determine whether the limit exists.

$\lim_{x\to 3^-}f(x)=\lim_{x\to 3^-}\frac{x^2-4}{x-2}=\lim_{x\to 3^-}\frac{(x+2)(x-2)}{x-2}=\frac{(5)(1)}{1}=5$

$\lim_{x\to 3^+}f(x)=\lim_{x\to 3^+}(2x-1)=2(6)-1=5$

Since the one-sided limits are equal, $\lim_{x\to 3}f(x)$ exists and $\lim_{x\to 3}f(x)=5$

However, note that: $5=\lim_{x\to 3}f(x)\neq f(3)=6$

i.e., $\lim_{x\to 3}f(x)\neq f(3)$

Hence, $f(x)$ is NOT continuous at $x=3$

5. $f(x) = \frac{x^2-1}{x^2-x-6}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - x - 6 =$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = -2$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to-2^-} \frac{x^2-1}{x^2-x-6} = \lim_{x\to-2^-} \frac{x^2-1}{(x+2)(x-3)} = \frac{3}{(-\varepsilon)(-5)} = \frac{3}{(\varepsilon)(5)} = \frac{\left(\frac{3}{5}\right)}{\varepsilon} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x + 2 < 0 \end{array}}$$

$$\lim_{x\to-2^+} \frac{x^2-1}{x^2-x-6} = \lim_{x\to-2^+} \frac{x^2-1}{(x+2)(x-3)} = \frac{3}{(\varepsilon)(-5)} = -\frac{\left(\frac{3}{5}\right)}{\varepsilon} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x + 2 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$$\lim_{x\to3^-} \frac{x^2-1}{x^2-x-6} = \lim_{x\to3^-} \frac{x^2-1}{(x+2)(x-3)} = \frac{8}{(5)(-\varepsilon)} = \frac{\left(\frac{8}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x - 3 < 0 \end{array}}$$

$$\lim_{x\to3^+} \frac{x^2-1}{x^2-x-6} = \lim_{x\to3^+} \frac{x^2-1}{(x+2)(x-3)} = \frac{8}{(5)(\varepsilon)} = \frac{\left(\frac{8}{5}\right)}{(\varepsilon)} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x - 3 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-1}{x^2-x-6} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

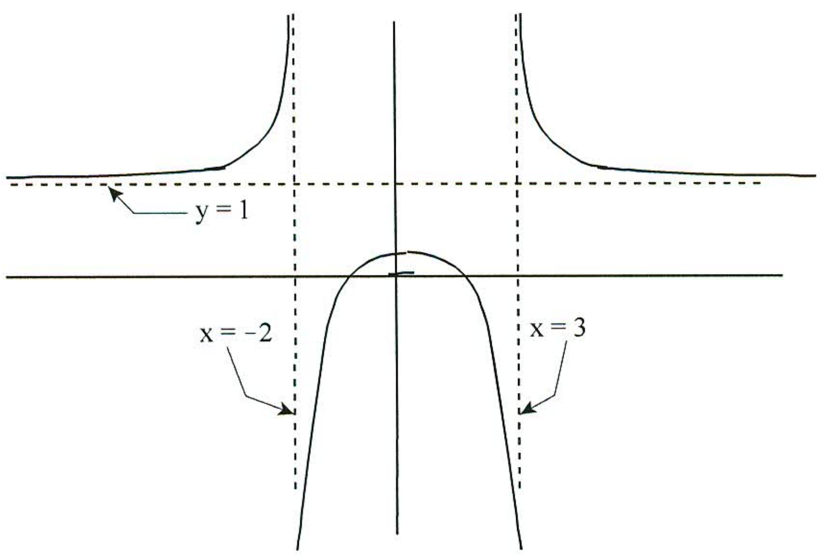
$\lim_{x\to+\infty} \frac{x^2-1}{x^2-x-6} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are finite and constant, $y = 1$ is a horizontal asymptote.

Summary:

$\lim_{x\to-2^-} \frac{x^2-1}{x^2-x-6} = +\infty$	
$\lim_{x\to-2^+} \frac{x^2-1}{x^2-x-6} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-1}{x^2-x-6} = 1$
$\lim_{x\to3^-} \frac{x^2-1}{x^2-x-6} = -\infty$	$\lim_{x\to+\infty} \frac{x^2-1}{x^2-x-6} = 1$
$\lim_{x\to3^+} \frac{x^2-1}{x^2-x-6} = +\infty$	

Graph $f(x) = \frac{x^2-1}{x^2-x-6}$



6. Compute: $\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} =$

Step #1 Try Plugging in:

$$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} = \frac{\sqrt{(2)+2}-2}{(2)-2} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Canceling:

$$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2}\cdot\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2} = \lim_{x\to 2}\frac{\left(\sqrt{x+2}\right)^2-(2)^2}{(x-2)\left[\sqrt{x+2}+2\right]}$$

1. $= \lim_{x\to 2}\frac{(x+2)-4}{(x-2)\left[\sqrt{x+2}+2\right]} = \lim_{x\to 6}\frac{(x-2)}{(x-2)\left[\sqrt{x+2}+2\right]} = \lim_{x\to 6}\frac{1}{\left[\sqrt{x+2}+2\right]}$

$= \frac{1}{\left[\sqrt{(2)+2}+2\right]} = \frac{1}{[2+2]} = \frac{1}{4}$

i.e., $\lim_{x\to 2}\frac{\sqrt{x+2}-2}{x-2} = \frac{1}{4}$

7.

$x =$	$f(x) =$
2.5	15.1
2.9	227.8
2.99	1212.3
2.999	21156.3
2.9999	834561.9

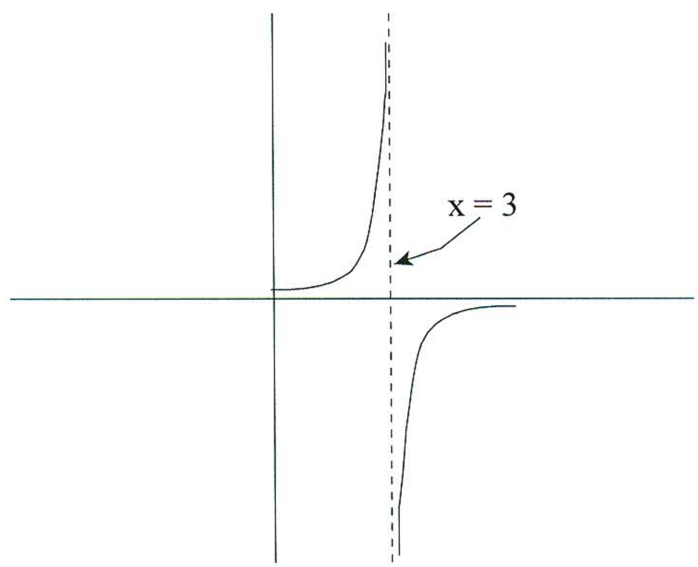
$x =$	$f(x) =$
3.5	−15.1
3.1	−227.8
3.01	−1212.3
3.001	−21156.3
3.0001	−834561.9

Based on the information in the table above, do the following:

(a) $\lim_{x\to 3^-} f(x) = +\infty$

(b) $\lim_{x\to 3^+} f(x) = -\infty$

(c) Graph $f(x)$



8. Compute: $\lim_{x\to -\infty} \frac{9x^5+4x^2-8x}{3x^4-8x-5} =$

$\lim_{x\to -\infty} \frac{9x^5+4x^2-8x}{3x^4-8x-5} = \lim_{x\to -\infty} \frac{9x^5}{3x^4} = \lim_{x\to -\infty} 3x = -\infty$

i.e., $\lim_{x\to -\infty} \frac{9x^5+4x^2-8x}{3x^4-8x-5} = -\infty$

Extra (5 pts - WOW!)

Compute: $\lim_{x\to\infty} \frac{\cos(x)+3}{2x} =$ (Justify your answer)

We cant't do this one directly. We have to make the following observations:

$-1 \leq \cos(x) \leq 1$

$\Rightarrow -1+3 \leq \cos(x)+3 \leq 1+3$

i.e., $2 \leq \cos(x)+3 \leq 4$

$\Rightarrow \frac{2}{2x} \leq \frac{\cos(x)+3}{2x} \leq \frac{4}{2x}$

i.e., $\frac{1}{x} \leq \frac{\cos(x)+3}{2x} \leq \frac{2}{x}$

$\Rightarrow \lim_{x\to\infty} \frac{1}{x} \leq \lim_{x\to\infty} \frac{\cos(x)+3}{2x} \leq \lim_{x\to\infty} \frac{2}{x}$

i.e., $\underbrace{\lim_{x\to\infty} \frac{1}{x}}_{=0} \leq \lim_{x\to\infty} \frac{\cos(x)+3}{2x} \leq \underbrace{\lim_{x\to\infty} \frac{2}{x}}_{=0}$

$\Rightarrow 0 \leq \lim_{x\to\infty} \frac{\cos(x)+3}{2x} \leq 0$

Hence, $\lim_{x\to\infty} \frac{\cos(x)+3}{2x} = 0$