Number Theory - Test #1

SUMMER 2023

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Instructions

Show CLEARLY how you arrive at you answers.

You can look in your text for reference (Statements of theorems, definitions, etc.)

Do not search the internet, or consult with others, for solutions (other than, perhaps, my own website)

1. State **Theorem 2.9**

2. Define the **prime** and **composite**

3. State **Theorem 3.1**

4. State **Corollary 1** (p. 40)

5. State **Theorem 3.2 (**the **Fundamental Theorem of Arithmetic)**

6. State the Corollary on page 42

7. Prove: $\sqrt{2}$ is irrational.

8. Prove: There are infinitely many primes

9. Prove: For any $n \in \mathbb{N}$, there exists a sequence of n consecutive composite numbers

10. Prove: $\{3, 5, 7\}$ is the only sequence of three consecutive odd prime numbers.

11. A local high school sells season tickets to their home football games. The season tickets are sold as single season tickets ($60 apiece) or in pars of season tickets ($105 per pair, so that both Mom and Dad can watch "Junior"). If total sales from season tickets amounts to $41,640, how many single season tickets and season ticket pairs have been sold? (Assume that at least one of each are sold. There are about 100 possibilities. List the "general formula." In addition, list the first three (x, y) pairs, as well as the last three (x, y) pairs.)

12. Prove: If $p \neq 5$ is an odd prime number, then either $p^2 - 1$ or $p^2 + 1$ is divisible by 10

13. Prove: A positive integer $a > 1$ is a perfect square if and only if, in the canonical form of a, all of the exponents of the primes are even integers.