MTH 1125 Test #1 - Solutions - (10 am class)

Spring 2010

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2+1}{x+4} =$

   (a) 1. Try Plugging In:

   $\lim_{x\to 2}\frac{x^2+1}{x+4} = \frac{(2)^2+1}{(2)+4} = \frac{5}{6}$

   i.e., $\lim_{x\to 2}\frac{x^2+1}{x+4} = \frac{5}{6}$

2. Compute: $\lim_{x\to 2}\frac{x^2-x-2}{x^2-4} =$

   (a) 1. Try Plugging In:

   $\lim_{x\to 2}\frac{x^2-x-2}{x^2-4} = \lim_{x\to 2}\frac{(2)^2-(2)-2}{(2)^2-4} = \frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Canceling:

   $\lim_{x\to 2}\frac{x^2-x-2}{x^2-4} = \lim_{x\to 2}\frac{(x+1)(x-2)}{(x-2)(x+2)} = \lim_{x\to 2}\frac{(x+1)}{(x+2)} = \frac{((2)+1)}{((2)+2)} = \frac{3}{4}$

   i.e., $\lim_{x\to 2}\frac{x^2-x-2}{x^2-4} = \frac{3}{4}$

3. Compute: $\lim_{x\to -1}\frac{x^2+1}{x^2-x-2} =$

   (a) 1. Try Plugging in:

   $\lim_{x\to -1}\frac{x^2+1}{x^2-x-2} = \lim_{x\to -1}\frac{(-1)^2+1}{(-1)^2-(-1)-2} = \frac{2}{0}$ No Good - Zero Divide!

   2. Try Factoring and Canceling:

   No good! This only works when Plugging In yields $\frac{0}{0}$

3. Compute the one-sided limits:

$$\lim_{x\to -1^-} \frac{x^2+1}{x^2-x-2} = \lim_{x\to -1^-} \frac{x^2+1}{(x+1)(x-2)} = \frac{2}{(-\varepsilon)(-3)} = \frac{2}{(\varepsilon)(3)} = \frac{\left(\frac{2}{3}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{ll} & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x+1 < 0 \end{array}$$

$$\lim_{x\to -1^+} \frac{x^2+1}{x^2-x-2} = \lim_{x\to -1^+} \frac{x^2+1}{(x+1)(x-2)} = \frac{2}{(+\varepsilon)(-3)} = \frac{\left(-\frac{2}{3}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{ll} & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x+1 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to -1} \frac{x^2+1}{x^2-x-2}$ **Does Not Exist!**

4. $f(x) = \begin{cases} \frac{x^3-27}{x-3} & \text{for } x < 3 \\ 6x+9 & \text{for } x \geq 3 \end{cases}$ Determine whether or not $f(x)$ is continuous at the point $x = 3$. (Justify your answer.)

If $f(x)$ is continuous at the point $x = 3$, then $\lim_{x\to 3} f(x) = f(3)$.

To see if this is true, we'll compute $\lim_{x\to 3} f(x)$.

Since the definiton of $f(x)$ changes at $x = 3$, we must compute the one-sided limits in order to determine whether the limit exists.

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^3-27}{x-3} = \lim_{x\to 3^-} \frac{(x-3)\left(x^2+3x+9\right)}{x-3}$$

$$= \lim_{x\to 3^-} (x^2+3x+9) = (3)^2 + 3(3) + 9 = 27$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (6x+9) = 6(3) + 9 = 27$$

Since the one-sided limits are equal, $\lim_{x\to 3} f(x)$ exists and is equal to the common value of the one-sided limits. i.e., $\lim_{x\to 3} f(x) = 27$

Furthermore, $f(3) = 6(3) + 9 = 27$.

$\Rightarrow \lim_{x\to 3} f(x) = f(3)$.

Since $\lim_{x\to 3} f(x) = f(3)$, $f(x)$ IS continuous at $x = 3$

5. $f(x) = \frac{x-3}{x+2}$ Find the asymptotes and graph

Verticals	Find all x-values that cause division by zero.

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$ possible vertical asymptote.

Compute the one-sided limits as $x \to -2$

$\lim_{x\to-2^-} \frac{x-3}{x+2} = \lim_{x\to-2^-} \frac{-5}{(-\varepsilon)} = +\infty$

(a)
$$\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x + 2 < 0 \end{array}$$

$\lim_{x\to-2^+} \frac{x-3}{x+2} = \lim_{x\to-2^+} \frac{-5}{(+\varepsilon)} = -\infty$

$$\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x + 2 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

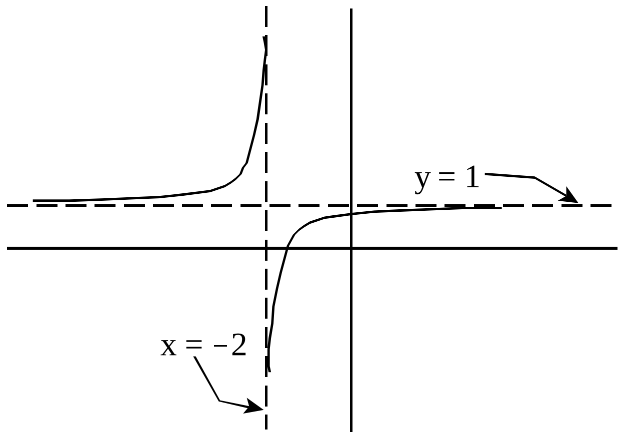
Horizontals	Compute limits as $x \to \pm\infty$

$\lim_{x\to-\infty} \frac{x-3}{x+2} = \lim_{x\to-\infty} \frac{x}{x} = \lim_{x\to-\infty} (1) = 1$

$\lim_{x\to+\infty} \frac{x-3}{x+2} = \lim_{x\to+\infty} \frac{x}{x} = \lim_{x\to+\infty} (1) = 1$

Since the limits are finite constants, $y = 1$ *IS a horizontal asymptote.*

Graph $f(x) = \frac{x-3}{x+2}$



6. Compute: $\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} =$

(a) 1. Try Plugging in:

$\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} = \lim_{x\to 1} \frac{\sqrt{5-(1)}-2}{(1)-1} = \frac{0}{0}$ No Good - Zero Divide!

2. Try Factoring and Canceling:

$$\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} = \lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} \cdot \frac{\sqrt{5-x}+2}{\sqrt{5-x}+2} = \lim_{x\to 1} \frac{\left(\sqrt{5-x}\right)^2-(2)^2}{(x-1)\left[\sqrt{5-x}+2\right]}$$

$$= \lim_{x\to 1} \frac{(5-x)-4}{(x-1)\left[\sqrt{5-x}+2\right]} = \lim_{x\to 1} \frac{(1-x)}{(x-1)\left[\sqrt{5-x}+2\right]} = \lim_{x\to 1} \frac{-(x-1)}{(x-1)\left[\sqrt{5-x}+2\right]}$$

$$= \lim_{x\to 1} \frac{-1}{\left[\sqrt{5-x}+2\right]} = \frac{-1}{\left[\sqrt{5-(1)}+2\right]} = \frac{-1}{\left[\sqrt{4}+2\right]} = \frac{-1}{[2+2]} = -\frac{1}{4}$$

i.e., $\lim_{x\to 1} \frac{\sqrt{5-x}-2}{x-1} = -\frac{1}{4}$

7. $f(x) = x^2+3x+4$; Compute $f'(x)$ **using the definition of derivative.** (i.e. compute $f'(x)$ using the "limit process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2+3(x+\Delta x)+4\right]-\left[x^2+3x+4\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[x^2+2x\Delta x+\Delta x^2+3x+3\Delta x+4\right]-\left[x^2+3x+4\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x+\Delta x^2+3\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(2x+\Delta x+3)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x + 3) = 2x + (0) + 3 = 2x + 3$

$\boxed{\text{i.e., } f'(x) = 2x + 3}$

8. Compute: $\lim_{x\to\infty} \frac{3x^4+4x^2-8x}{6x^5+3x^4-8x-5} =$

$$\underbrace{\lim_{x\to\infty} \frac{3x^4+4x^2-8x}{6x^5+3x^4-8x-5} = \lim_{x\to\infty} \frac{3x^4}{6x^5}}_{\text{Terms of highest dgree dominate}} = \lim_{x\to\infty} \frac{1}{2x} = 0$$

$\boxed{\text{i.e., } \lim_{x\to\infty} \frac{3x^4+4x^2-8x}{6x^5+3x^4-8x-5} = 0}$