MTH 1125 Test #1 - (2 pm class) - Solutions

Fall 2023

Pat Rossi Name ______________________

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2+3x-8}{x^2+3x+5} =$

Step #1 Try Plugging In:

$\lim_{x\to 2}\frac{x^2+3x-8}{x^2+3x+5} = \frac{(2)^2+3(2)-8}{(2)^2+3(2)+5} = \frac{2}{15}$

i.e., $\lim_{x\to 2}\frac{x^2+3x-8}{x^2+3x+5} = \frac{2}{15}$

2. Compute: $\lim_{x\to 2}\frac{x^2-7x+10}{2x^2-3x-2} =$

Step #1 Try Plugging In:

$\lim_{x\to 2}\frac{x^2-7x+10}{2x^2-3x-2} = \frac{(2)^2-7(2)+10}{2(2)^2-3(2)-2} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 2}\frac{x^2-7x+10}{2x^2-3x-2} = \lim_{x\to 2}\frac{(x-2)(x-5)}{(x-2)(2x+1)} = \lim_{x\to 2}\frac{(x-5)}{(2x+1)} = \frac{(2)-5}{2(2)+1} = \frac{-3}{5} = -\frac{3}{5}$

i.e., $\lim_{x\to 2}\frac{x^2-7x+10}{2x^2-3x-2} = -\frac{3}{5}$

3. Compute: $\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+x-6} =$

Step #1 Try Plugging in:

$\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+x-6} = \frac{(2)^2+2(2)-9}{(2)^2+(2)-6} = \frac{-1}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$\lim_{x\to\ 2^-} \frac{x^2+2x-9}{x^2+x-6} = \lim_{x\to\ 2^-} \frac{x^2+2x-9}{(x+3)(x-2)} = \frac{-1}{(5)(-\varepsilon)} = \frac{1}{(5)(\varepsilon)} = \frac{\left(\frac{1}{5}\right)}{(\varepsilon)} = +\infty$

$x \to 2^-$
$\Rightarrow \quad x < 2$
$\Rightarrow \quad x - 2 < 0$

$\lim_{x\to\ 2^+} \frac{x^2+2x-9}{x^2+x-6} = \lim_{x\to\ 2^+} \frac{x^2+2x-9}{(x+3)(x-2)} = \frac{-1}{(5)(+\varepsilon)} = \frac{\left(-\frac{1}{5}\right)}{(\varepsilon)} = -\infty$

$x \to 2^+$
$\Rightarrow \quad x > 2$
$\Rightarrow \quad x - 2 > 0$

Since the one-sided limits are not equal, $\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+x-6}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{4x^8+3x-3}{4x^6+7x^5-5x} = \lim_{x\to\ -\infty} \frac{4x^8}{4x^6} = \lim_{x\to\ -\infty} x^2 = +\infty$

i.e., $\lim_{x\to\ -\infty} \frac{4x^8+3x-3}{4x^6+7x^5-5x} = +\infty$

5. $f(x) = \frac{x^2-2x+3}{x^2+x-2}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x = -2$ and $x = 1$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -2^-} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to\ -2^-} \frac{x^2-2x+3}{(x+2)(x-1)} = \frac{11}{(-\varepsilon)(-3)} = \frac{11}{(\varepsilon)(3)} = \frac{\left(\frac{11}{3}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|} \hline & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \\ \hline \end{array}$$

$$\lim_{x\to\ -2^+} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to\ -2^+} \frac{x^2-2x+3}{(x+2)(x-1)} = \frac{11}{(+\varepsilon)(-3)} = \frac{\left(\frac{11}{-3}\right)}{\varepsilon} = \frac{\left(-\frac{11}{3}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{|ll|} \hline & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \\ \hline \end{array}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$$\lim_{x\to\ 1^-} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to\ 1^-} \frac{x^2-2x+3}{(x+2)(x-1)} = \frac{2}{(3)(-\varepsilon)} = \frac{\left(\frac{2}{3}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{|ll|} \hline & x \to 1^- \\ \Rightarrow & x < 1 \\ \hline \end{array}$$

$$\lim_{x\to\ 1^+} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to 1^+} \frac{x^2-2x+3}{(x+2)(x-1)} = \frac{2}{(3)(\varepsilon)} = \frac{\left(\frac{2}{3}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{|ll|} \hline & x \to 1^+ \\ \Rightarrow & x > 1 \\ \hline \end{array}$$

Since the one-sided limits are **infinite,** $x = 1$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

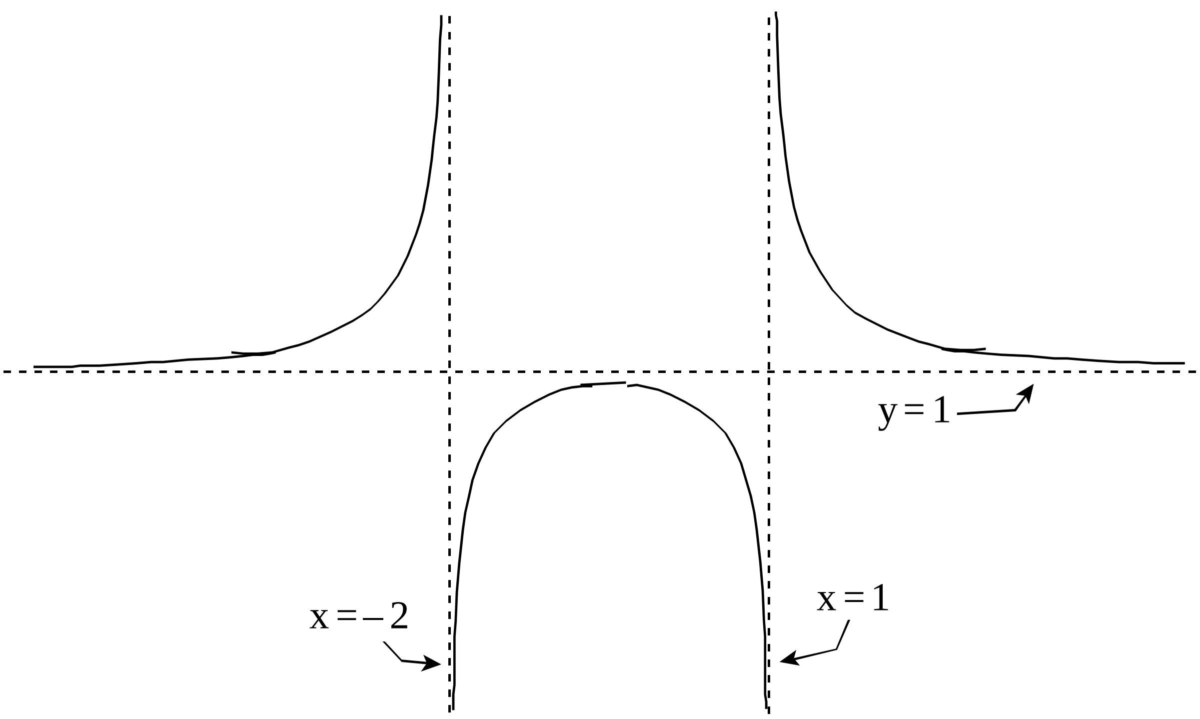
$\lim_{x\to+\infty} \frac{x^2-2x+3}{x^2+x-2} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to\ -2^-} \frac{x^2-2x+3}{x^2+x-2} = +\infty$	
$\lim_{x\to\ -2^+} \frac{x^2-2x+3}{x^2+x-2} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-2x+3}{x^2+x-2} = 1$
$\lim_{x\to\ 1^-} \frac{x^2-2x+3}{x^2+x-2} = -\infty$	$\lim_{x\to+\infty} \frac{x^2-2x+3}{x^2+x-2} = 1$
$\lim_{x\to\ 1^+} \frac{x^2-2x+3}{x^2+x-2} = +\infty$	

Graph $f(x) = \frac{x^2-2x+3}{x^2+x-2}$



6. Compute: $\lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9} =$

Step #1 Try Plugging in:

$$\lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9} = \frac{\sqrt{(9)+7}-4}{9-9} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9} = \lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9} \cdot \frac{\sqrt{x+7}+4}{\sqrt{x+7}+4} = \lim_{x\to 9} \frac{\left(\sqrt{x+7}\right)^2-(4)^2}{(x-9)\left[\sqrt{x+7}+4\right]}$$

$$= \lim_{x\to 9} \frac{(x+7)-16}{(x-9)\left[\sqrt{x+7}+4\right]} = \lim_{x\to 9} \frac{(x-9)}{(x-9)\left[\sqrt{x+7}+4\right]} = \lim_{x\to 9} \frac{1}{\left[\sqrt{x+7}+4\right]}$$

$$= \frac{1}{\left[\sqrt{(9)+7}+4\right]} = \frac{1}{[4+4]} = \frac{1}{8}$$

i.e., $\lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9} = \frac{1}{8}$

7.

$x =$	$f(x) =$
1.5	10
1.9	100
1.99	$1,000$
1.999	$10,000$
1.9999	$100,000$

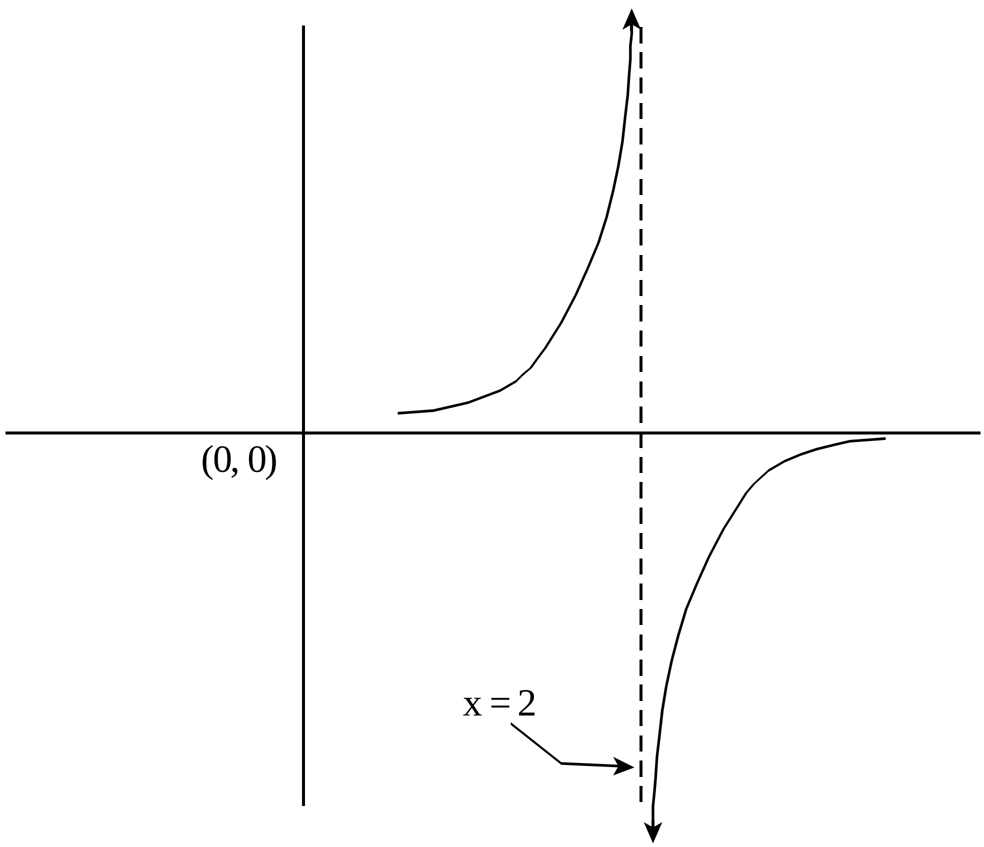
$x =$	$f(x) =$
2.5	-10
2.1	-100
2.01	$-1,000$
2.001	$-10,000$
2.0001	$-100,000$

Based on the information in the table above, compute/do the following:

(a) $\lim_{x\to 2^-} f(x) = \infty$

(b) $\lim_{x\to 2^+} f(x) = -\infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 2$. (Justify Your Answer)

$$f(x) = \begin{cases} 3x - 3 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \\ x^2 - 1 & \text{for } x > 2 \end{cases}$$

$f(x)$ is continuous at the point $x = 2$ exactly when $\lim_{x \to 2} f(x) = f(2)$

Since the definition of $f(x)$ changes at $x = 2$, we must compute the one-sided limits as x approaches 2, in order to compute $\lim_{x \to 2} f(x)$.

Observe: As $x \to 2^-$, $x < 2$.

Therefore: $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x - 3) = 3(2) - 3 = 3$

Also: As $x \to 2^+$, $x > 2$.

Therefore: $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - 1) = (2)^2 - 1 = 3$

Since the one-sided limits are equal, $\lim_{x \to 3} f(x)$ exists, and is equal to the common value of the one-sided limits.

i.e., $\lim_{x \to 2} f(x) = 3$

Finally, note that $f(2) = 3$

$\Rightarrow \lim_{x \to 2} f(x) = f(2)$

Hence, $f(x)$ is continuous at the point $x = 2$