# MTH 1125 - Test 2 (2pm Class) - Pod A - Solutions

FALL 2020

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[3x^5+3x^4+5x^3+6x^2+15x+24\sqrt{x}+6\right]=$

$$\frac{d}{dx}\left[3x^5+3x^4+5x^3+6x^2+15x+24\sqrt{x}+6\right]$$

$$=3\left[5x^4\right]+3\left[4x^3\right]+5\left[3x^2\right]+6\left[2x\right]+15+24\left[\tfrac{1}{2}x^{-\frac{1}{2}}\right]+0$$

$$=15x^4+12x^3+15x^2+12x+15+12x^{-\frac{1}{2}}$$

$$\text{i.e., } \frac{d}{dx}\left[3x^5+3x^4+5x^3+6x^2+15x+24\sqrt{x}+6\right]=15x^4+12x^3+15x^2+12x+15+12x^{-\frac{1}{2}}$$

2. Compute: $\frac{d}{dx}\left[\left(5x^2+\sec(x)\right)\left(2x^3+6x\right)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(5x^2+\sec(x)\right)}_{1^{st}}\underbrace{\left(2x^3+6x\right)}_{2^{nd}}\right]=\underbrace{\left(10x+\sec(x)\tan(x)\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\left(2x^3+6x\right)}_{2^{nd}}+\underbrace{\left(6x^2+6\right)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\left(5x^2+\sec(x)\right)}_{1^{st}}$$

$$\frac{d}{dx}\left[\left(5x^2+\sec(x)\right)\left(2x^3+6x\right)\right]=\left(10x+\sec(x)\tan(x)\right)\left(2x^3+6x\right)+\left(6x^2+6\right)\left(5x^2+\sec(x)\right)$$

3. Compute: $\frac{d}{dx}\left[\frac{3x^4+6x^2+9}{4x^2+1}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{3x^4+6x^2+9}^{\text{top}}}{\underbrace{4x^2+1}_{\text{Bottom}}}\right]=\frac{\overbrace{\left(12x^3+12x\right)}^{\text{top prime}}\cdot\overbrace{\left(4x^2+1\right)}^{\text{bottom}}-\overbrace{8x}^{\text{bottom prime}}\cdot\overbrace{\left(3x^4+6x^2+9\right)}^{\text{top}}}{\underbrace{\left(4x^2+1\right)^2}_{\text{bottom squared}}}$$

$$\text{i.e., } \frac{d}{dx}\left[\frac{3x^4+6x^2+9}{4x^2+1}\right]=\frac{\left(12x^3+12x\right)\left(4x^2+1\right)-8x\left(3x^4+6x^2+9\right)}{\left(4x^2+1\right)^2}$$

4. Compute: $\frac{d}{dx}\left[\left(6x^3+9x^2+4x\right)^6\right]=$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(6x^3+9x^2+4x\right)^6\right]=\underbrace{6\left(6x^3+9x^2+4x\right)^5}_{\substack{\text{power rule}\\ \text{as usual}}}\cdot\underbrace{\left(18x^2+18x+4\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(6x^3+9x^2+4x\right)^6\right]=6\left(6x^3+9x^2+4x\right)^5\left(18x^2+18x+4\right)$

5. Given that $f(x)=2x^2+x-5$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2,5)$.

We need two things:

i. A **point** on the line (We have that: $(x_1,y_1)=(2,5)$)

ii. The **slope** of the line (This is $f'(x_1)$)

$f'(x)=4x+1$

At the point $(x_1,y_1)=(2,5)$, **the slope is** $f'(2)=4(2)+3=11$

We will use the Point-Slope equation of a line:

$(y-y_1)=m(x-x_1)$ (Where $m$ is the slope and $(x_1,y_1)$ is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$(y-5)=11(x-2)$

The equation of the line tangent is $(y-5)=11(x-2)$

6. Given that $w = 2x^2 + 3x$ and that $x = \sec(v)$; compute $\frac{dw}{dv}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

**We know:**

$\frac{dw}{dx} = 4x + 3$

$\frac{dx}{dv} = \sec(v)\tan(v)$

**We want:** $\frac{dw}{dv}$

By the Liebniz form of the Chain Rule:

$$\frac{dw}{dv} = \frac{dw}{dx}\frac{dx}{dv} = (4x+3)\sec(v)\tan(v) = \underbrace{(4\sec(v)+3)\sec(v)\tan(v)}_{\text{express solely in terms of independent variable } v}$$

| |
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| i.e. $\frac{dw}{dv} = (4\sec(v)+3)\sec(v)\tan(v)$ |

7. Compute: $\frac{d}{dx}\left[\sec\left(4x^3+6x^2\right)\right] =$

| | | |
|---|---|---|
| Outer: | = | $\sec(\quad)$ |
| Deriv. of outer | = | $\sec(\quad)\tan(\quad)$ |

$$\frac{d}{dx}\left[\underset{\substack{\uparrow\\ \text{outer}}}{\sec}\Big(\underset{\substack{\uparrow\\ \text{inner}}}{\underbrace{4x^3+6x^2}}\Big)\right] = \underbrace{\sec\left(4x^3+6x^2\right)\tan\left(4x^3+6x^2\right)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\left(12x^2+12x\right)}_{\text{deriv. of inner}}$$

| |
|---|
| i.e., $\frac{d}{dx}\left[\sec\left(4x^3+6x^2\right)\right] = \sec\left(4x^3+6x^2\right)\tan\left(4x^3+6x^2\right)\left(12x^2+12x\right)$ |

8. Compute: $\frac{d}{dx}\left[\left(\frac{5x^2+12x}{4x^2+8x}\right)^6\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{5x^2+12x}{4x^2+8x}\right)}_{(g(x))^n}^6\right] = \underbrace{6\left(\frac{5x^2+12x}{4x^2+8x}\right)^5}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{5x^2+12x}{4x^2+8x}\right]\right)}_{\substack{\text{deriv of}\\ \text{inner Function}}}$$

$$= 6\left(\frac{5x^2+12x}{4x^2+8x}\right)^5 \underbrace{\frac{(10x+12)\left(4x^2+8x\right) - (8x+8)\left(5x^2+12x\right)}{\left(4x^2+8x\right)^2}}_{\substack{\text{quotient}\\ \text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{5x^2+12x}{4x^2+8x}\right)^6\right] = 6\left(\frac{5x^2+12x}{4x^2+8x}\right)^5 \frac{(10x+12)\left(4x^2+8x\right) - (8x+8)\left(5x^2+12x\right)}{\left(4x^2+8x\right)^2}$

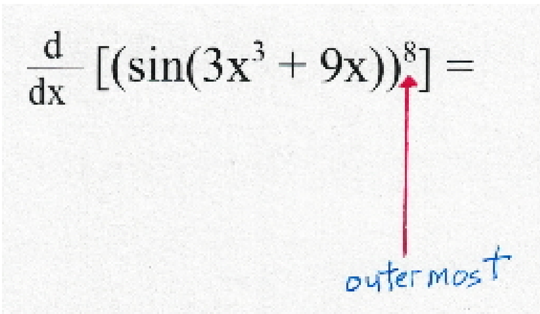
9. Compute: $\frac{d}{dx}\left[\sin^8\left(3x^3+9x\right)\right]=$

Let's rewrite this:

$\frac{d}{dx}\left[\left(\sin\left(3x^3+9x\right)\right)^8\right]$

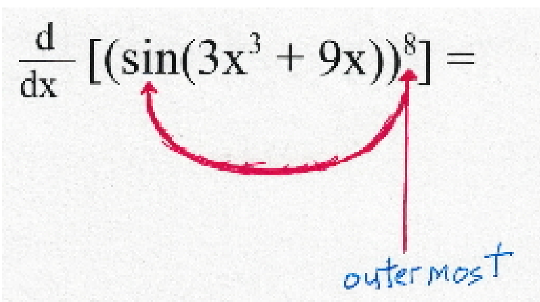
This is the composition of *three* functions.

Differentiate the outermost function and evaluate it at everything inside



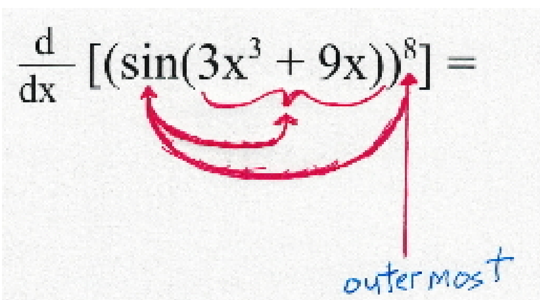

This yields: $8\left(\sin\left(3x^3+9x\right)\right)^7$

**Next:** Multiply by the derivative of the next outermost function and evaluate it at everything inside of it.




This yields: $8\left(\sin\left(3x^3+9x\right)\right)^7\cdot\cos\left(3x^3+9x\right)$

**Finally:** Multiply by the derivative of the innermost function.




This yields: $8\left(\sin\left(3x^3+9x\right)\right)^7\cos\left(3x^3+9x\right)\cdot\left(9x^2+9\right)$

i.e., $\frac{d}{dx}\left[\left(\sin^{8}\left(3x^{3}+9x\right)\right)\right] = 8\left(\sin\left(3x^{3}+9x\right)\right)^{7}\cos\left(3x^{3}+9x\right)\cdot\left(9x^{2}+9\right)$

**Alternatively:**

$\frac{d}{dx}\left[\left(\sin^{8}\left(3x^{3}+9x\right)\right)\right]$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE. Re-write!

$\frac{d}{dx}\left[\left(\sin\left(3x^{3}+9x\right)\right)^{8}\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[\left(\sin\left(3x^{3}+9x\right)\right)^{8}\right] = \underbrace{8\left(\sin\left(3x^{3}+9x\right)\right)^{7}}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\sin\left(3x^{3}+9x\right)\right]\right)}_{\text{derivative of inner}}$$

$$= 8\left(\sin\left(3x^{3}+9x\right)\right)^{7} \cdot \underbrace{\left[\cos\left(3x^{3}+9x\right)\cdot\left(9x^{2}+9\right)\right]}_{\text{Chain Rule}}$$

i.e., $\frac{d}{dx}\left[\left(\sin^{8}\left(3x^{3}+9x\right)\right)\right] = 8\left(\sin\left(3x^{3}+9x\right)\right)^{7}\cos\left(3x^{3}+9x\right)\left(9x^{2}+9\right)$

10. Given that $x^4 + x^3y^4 = \cos(y)$, compute $\frac{dy}{dx}$

i. Differentiate both sides w.r.t. $x$

$$\frac{d}{dx}\left[x^4 + \underbrace{x^3}_{1^{\text{st}}}\underbrace{y^4}_{2^{\text{nd}}}\right] = \frac{d}{dx}\left[\cos(y)\right]$$

$$\Rightarrow 4x^3 + \left(\underbrace{3x^2}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^4}_{2^{\text{nd}}} + \underbrace{4y^3\frac{dy}{dx}}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{x^3}_{1^{\text{st}}}\right) = -\sin(y) \cdot \frac{dy}{dx}$$

Simplifying, we have:

$$4x^3 + 3x^2y^4 + 4x^3y^3\frac{dy}{dx} = -\sin(y)\frac{dy}{dx}$$

ii. Solve algebraically for $\frac{dy}{dx}$

a. Get $\frac{dy}{dx}$ terms on left side, all other terms on right side

$$\Rightarrow 4x^3y^3\frac{dy}{dx} + \sin(y)\frac{dy}{dx} = -4x^3 - 3x^2y^4$$

b. Factor out $\frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx}\left(4x^3y^3 + \sin(y)\right) = -4x^3 - 3x^2y^4$$

c. Divide both sides by the cofactor of $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-4x^3 - 3x^2y^4}{4x^3y^3 + \sin(y)} = -\frac{4x^3 + 3x^2y^4}{4x^3y^3 + \sin(y)}$$

$$\boxed{\frac{dy}{dx} = \frac{-4x^3 - 3x^2y^4}{4x^3y^3 + \sin(y)} = -\frac{4x^3 + 3x^2y^4}{4x^3y^3 + \sin(y)}}$$

11. Given that $f(x) = 3x^2 - 6x + 5$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[3(x+\Delta x)^2 - 6(x+\Delta x) + 5\right] - \left[3x^2 - 6x + 5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[3\left(x^2 + 2x\Delta x + \Delta x^2\right) - 6(x+\Delta x) + 5\right] - \left[3x^2 - 6x + 5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[3x^2 + 6x\Delta x + 3\Delta x^2 - 6x - 6\Delta x + 5\right] - \left[3x^2 - 6x + 5\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2 - 6\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(6x + 3\Delta x - 6)}{\Delta x} = \lim_{\Delta x \to 0} (6x + 3\Delta x - 6) = 6x + 3(0) - 6 = 6x - 6$$

i.e., $f'(x) = 6x - 6$

**Extra** (Wow! 10 Points)

Given that $L'(x) = \frac{1}{x}$ (i.e., $\frac{d}{dx}[L(x)] = \frac{1}{x}$); compute $\frac{d}{dx}\left[L\left(x^2\right)\right]$

| | | |
|---|---|---|
| Outer: | $=$ | $L(\ \ )$ |
| Deriv. of outer | $=$ | $\frac{1}{(\ \ )}$ |

$$\underset{\text{outer}}{\frac{d}{dx}\Big[L}\Big(\underbrace{x^2}_{\text{inner}}\Big)\Big] = \underbrace{\frac{1}{(x^2)}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{2x}_{\text{deriv. of inner}} = \frac{2x}{x^2} = \frac{2}{x}$$

i.e., $\frac{d}{dx}\left[L\left(x^2\right)\right] = \frac{2x}{x^2} = \frac{2}{x}$