Mth 1125 Test #1 - Solutions

Fall 2003

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Instructions. Show clearly how you arrive at your answers.

1. Compute: $\lim_{x\to 1}\frac{x^2+x-2}{x^2-4x+3}=$

   Step #1: Try plugging in: $\lim_{x\to 1}\frac{x^2+x-2}{x^2-4x+3}=\frac{1^2+1-2}{1^2-4(1)+3}=\frac{0}{0}$ no good - Zero divide

   Step #2: Cancel common factors, THEN plug in: $\lim_{x\to 1}\frac{x^2+x-2}{x^2-4x+3}=\lim_{x\to 1}\frac{(x-1)(x+2)}{(x-1)(x-3)}=\lim_{x\to 1}\frac{(x+2)}{(x-3)}=\frac{(1+2)}{(1-3)}=-\frac{3}{2}$

   i.e., $\lim_{x\to 1}\frac{x^2+x-2}{x^2-4x+3}=-\frac{3}{2}$

2. Compute: $\lim_{x\to 2}\frac{2x^3-3x^2+7}{x^3-3}=$

   Step #1: Try plugging in: $\lim_{x\to 2}\frac{2x^3-3x^2+7}{x^3-3}=\frac{2(2)^3-3(2)^2+7}{(2)^3-3}=\frac{11}{5}$

   i.e., $\lim_{x\to 2}\frac{2x^3-3x^2+7}{x^3-3}=\frac{11}{5}$

3. Compute: $\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}=$

   Step #1: Try plugging in: $\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}=\frac{\sqrt{7+2}-3}{2-2}=\frac{0}{0}$ no good - Zero divide

   Step #2: Cancel common factors, THEN plug in:
   $\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}=\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}\cdot\frac{\sqrt{7+x}+3}{\sqrt{7+x}+3}=\lim_{x\to 2}\frac{(7+x)-9}{(x-2)\left(\sqrt{7+x}+3\right)}=\lim_{x\to 2}\frac{(x-2)}{(x-2)\left(\sqrt{7+x}+3\right)}$
   $=\lim_{x\to 2}\frac{1}{\left(\sqrt{7+2}+3\right)}=\frac{1}{6}$

   i.e., $\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}=\frac{1}{6}$

4. $\lim_{x\to\infty}\frac{5x^3-2x+5}{2x^3-4x^2+8x-2}=$

   As $x\to\infty$, terms of highest degree in numerator and denominator dominate the other terms> So:

   $\lim_{x\to\infty}\frac{5x^3-2x+5}{2x^3-4x^2+8x-2}=\lim_{x\to\infty}\frac{5x^3}{2x^3}=\lim_{x\to\infty}\frac{5}{2}=\frac{5}{2}$

   i.e., $\lim_{x\to\infty}\frac{5x^3-2x+5}{2x^3-4x^2+8x-2}=\frac{5}{2}$

5. Find asymptotes and graph: $f(x) = \frac{2x^3+3}{x+2}$

Verticals: find x-values that cause division by zero:

$x = -2$ causes division by zero.

$\lim_{x \to -2^-} \frac{2x^3+3}{x+2} = \frac{-13}{-\varepsilon} = +\infty$

$$\begin{aligned} & x \to -2^- \\ \Rightarrow\ & x < -2 \\ \Rightarrow\ & x + 2 < 0 \end{aligned}$$

$\lim_{x \to -2^-} \frac{2x^3+3}{x+2} = \frac{-13}{\varepsilon} = -\infty$

$$\begin{aligned} & x \to -2^+ \\ \Rightarrow\ & x > -2 \\ \Rightarrow\ & x + 2 > 0 \end{aligned}$$

Infinite limits tell us that $x = -2$ IS a vertical asymptote.

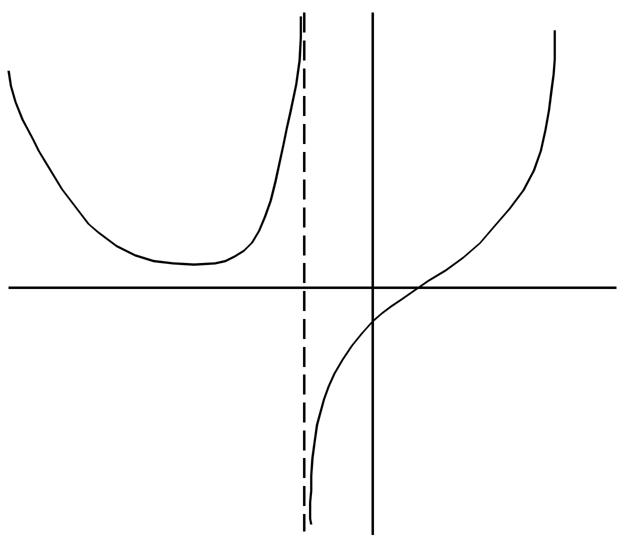
Horizontals: Let $x \to -\infty$ and $x \to \infty$

$\lim_{x \to -\infty} \frac{2x^3+3}{x+2} = \lim_{x \to -\infty} \frac{2x^3}{x} = \lim_{x \to -\infty} 2x^2 = \infty$

$\lim_{x \to \infty} \frac{2x^3+3}{x+2} = \lim_{x \to \infty} \frac{2x^3}{x} = \lim_{x \to \infty} 2x^2 = \infty$

Limits are not constant. There are no horizontal asymptotes.

Graph: $f(x) = \frac{2x^3+3}{x+2}$



6. $\lim_{x \to 3} \frac{x^2+3}{x^2-9} =$

Step #1: Try plugging in: $\lim_{x \to 3} \frac{x^2+3}{x^2-9} = \frac{12}{0}$ zreo divide - no good

Step #2: this won't work for limits of the form: $\frac{\text{non-zero}}{\text{zero}}$

Step #3: Consider the one-sided limits:

$\lim_{x \to 3^-} \frac{x^2+3}{x^2-9} = \frac{12}{-\varepsilon} = -\infty$

$\lim_{x \to 3^+} \frac{x^2+3}{x^2-9} = \frac{12}{+\varepsilon} = +\infty$

Since the one-sided limits are not equal, $\lim_{x \to 3} \frac{x^2+3}{x^2-9}$ Does Not Exist.

7. Given:

$x =$	$f(x)$
1.000	-3.1
2.000	-45.3
2.500	-559.2
2.900	-8547.3
2.990	-99131.8

$x =$	$f(x)$
5.000	3.5
4.000	85.1
3.500	759.2
3.100	8412.7
3.010	95927.2

determine:

(a) $\lim_{x \to 3^-} f(x) = -\infty$

(b) $\lim_{x \to 3^+} f(x) = +\infty$

(c) Sketch a rough graph of $f(x)$.