MTH 1125 12pm Class - Test #4 -Solutions

Fall 2021

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Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int (12x^3 + 9x^2 - 6x + 6\sqrt{x} + 4)\, dx =$

$$\int (12x^3 + 9x^2 - 6x + 6\sqrt{x} + 4)\, dx = \int \left(12x^3 + 9x^2 - 6x + 6x^{\frac{1}{2}} + 4\right) dx$$

$$= 12\left[\tfrac{x^4}{4}\right] + 9\left[\tfrac{x^3}{3}\right] - 6\left[\tfrac{x^2}{2}\right] + 6\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + 4x + C = 3x^4 + 3x^3 - 3x^2 + 4x^{\frac{3}{2}} + 4x + C$$

i.e., $\int (12x^3 + 9x^2 - 6x + 6\sqrt{x} + 4)\, dx = 3x^4 + 3x^3 - 3x^2 + 4x^{\frac{3}{2}} + 4x + C$

2. **Compute:** $\int \left(6x^2 + 16x + 5\right)^9 (3x+4)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

      Yes! $\left(6x^2 + 16x + 5\right)^9$ (A function raised to a power is *always* a composite function!)

      Let $u =$ the "inner" of the composite function

      $\Rightarrow u = (6x^2 + 16x + 5)$

   b. Is there an (approximate) function/derivative pair?

      Yes! $\underbrace{(6x^2 + 16x + 5)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+4)}_{\text{deriv}}$

      Let $u =$ the "function" of the function/deriv pair

      $\Rightarrow u = (6x^2 + 16x + 5)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

      (i.e., do criteria **a** and **b** suggest the same choice of u ?)

      Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2 + 16x + 5 \\ \Rightarrow \tfrac{du}{dx} &= 12x + 16 \\ \Rightarrow du &= (12x+16)\,dx \\ \Rightarrow \tfrac{1}{4}du &= (3x+4)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2 + 16x + 5\right)^9}_{u^9} \underbrace{(3x+4)\,dx}_{\frac{1}{4}du} = \int u^9 \tfrac{1}{4} du = \tfrac{1}{4} \int u^9 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4} \int u^9 du = \tfrac{1}{4}\left[\tfrac{u^{10}}{10}\right] + C = \tfrac{1}{40} u^{10} + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2 + 16x + 5\right)^9 (3x+4)\,dx = \underbrace{\frac{1}{40}\left(6x^2 + 16x + 5\right)^{10} + C}_{\frac{1}{40}u^{10} + C}$$

i.e., $\int \left(6x^2+16x+5\right)^9 (3x+4)\,dx = \frac{1}{40}\left(6x^2+16x+5\right)^{10}+C$

3. **Compute:** $\int (4\sin(x) - 5\csc^2(x) + 6\csc(x)\cot(x))\,dx =$

$\int (4\sin(x) - 5\csc^2(x) + 6\csc(x)\cot(x))\,dx = 4\left[-\cos(x)\right] - 5\left[-\cot(x)\right] + 6\left[-\csc(x)\right] + C$

$= -4\cos(x) + 5\cot(x) - 6\csc(x) + C$

i.e., $\int (4\sin(x) - 5\csc^2(x) + 6\csc(x)\cot(x))\,dx = -4\cos(x) + 5\cot(x) - 6\csc(x) + C$

4. **Compute:** $\int \cos\left(4x^2+6x+3\right)(4x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(4x^2+6x+3\right)$

   $\nearrow$ outer $\quad\nwarrow$ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(4x^2+6x+3\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(4x^2+6x+3\right)}_{\text{function}} - - - - \rightarrow \underbrace{(4x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(4x^2+6x+3\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 4x^2+6x+3 \\ \Rightarrow \frac{du}{dx} &= 8x+6 \\ \Rightarrow du &= (8x+6)\,dx \\ \Rightarrow \tfrac{1}{2}du &= (4x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(4x^2+6x+3\right)}_{\cos(u)}\underbrace{(4x+3)\,dx}_{\frac{1}{2}du} = \int \cos(u)\,\tfrac{1}{2}du = \tfrac{1}{2}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{2}\int \cos(u)\,du = \tfrac{1}{2}\left[\sin(u)\right] + C = \tfrac{1}{2}\sin(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \cos\left(4x^2+6x+3\right)(4x+3)\,dx = \underbrace{\frac{1}{2}\sin\left(4x^2+6x+3\right) + C}_{\frac{1}{2}\sin(u)+C}$$

i.e., $\int \cos\left(4x^2+6x+3\right)(4x+3)\,dx = \frac{1}{2}\sin\left(4x^2+6x+3\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(6x^2+6x+2\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(6x^2+6x+2\right)}_{f(x)}dx = \underbrace{\left[6\left(\frac{x^3}{3}\right)+6\left(\frac{x2}{2}\right)+2x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[2x^3+3x^2+2x\right]_{-1}^{1}}$$

$$= \underbrace{\left[2(1)^3+3(1)^2+2(1)\right]}_{F(1)} - \underbrace{\left[2(-1)^3+3(-1)^2+2(-1)\right]}_{F(-1)}$$

$$= 7-(-1) = 8$$

i.e., $\int_{-1}^{1}\left(6x^2+6x+2\right)dx = 8$

6. **Compute:** $\int_{x=0}^{x-1} \left(x^4+1\right)^4 x^3 \; dx =$ (The answer may not be a whole number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(x^4+1\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (x^4+1)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(x^4+1)}_{\text{function}} - - - - - \rightarrow \underbrace{x^3}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (x^4+1)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= x^4+1 \\ \Rightarrow \frac{du}{dx} &= 4x^3 \\ \Rightarrow du &= 4x^3 \; dx \\ \Rightarrow \frac{1}{4}du &= x^3 \; dx \end{aligned}$$

When $x = 0$, $u = x^4 + 1 = (0)^4 + 1 = 1$
When $x = 1$, $u = x^4 + 1 = (1)^4 + 1 = 2$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(x^3+1\right)^4}_{u^4} \underbrace{x^3 \; dx}_{\frac{1}{4}du} = \int_{u=1}^{u=2} u^4 \cdot \frac{1}{4}du = \frac{1}{4}\int_{u=1}^{u=2} u^4 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{4}\int_{u=1}^{u=2} u^4 du = \frac{1}{4}\left[\frac{u^5}{5}\right]_{u=1}^{u=2} = \left[\frac{u^5}{20}\right]_{u=1}^{u=2} = \underbrace{\frac{(2)^5}{20}}_{F(2)} - \underbrace{\frac{(1)^5}{20}}_{F(1)} = \frac{32}{20} - \left(\frac{1}{20}\right) = \frac{31}{20}$$

$$\text{i.e., } \int_0^1 \left(x^4+1\right)^4 x^3 \; dx = \frac{31}{20}$$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(2x^4+4x^2-8\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(2x^4+4x^2-8\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{2x^4+4x^2-8}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(8x^3+8x\right)}_{g'(x)} = \frac{8x^3+8x}{2x^4+4x^2-8} = \frac{4x^3+4x}{x^4+2x^2-4}$$

$$\text{i.e., } \frac{d}{dx}\left[\ln\left(2x^4+4x^2-8\right)\right] = \frac{8x^3+8x}{2x^4+4x^2-8} = \frac{4x^3+4x}{x^4+2x^2-4}$$

Extra! (Wow! - 5 pts. Can you believe it?) **Compute:** $\int \frac{2x+1}{6x^2+6x+15} dx =$

$$\int \frac{2x+1}{6x^2+6x+15} dx \underbrace{=}_{\text{re-write}} \int \frac{1}{6x^2+6x+15}\,(2x+1)\,dx$$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\frac{1}{6x^2+6x+15}$ is the same as $\left(6x^2+6x+15\right)^{-1}$, so it is a function raised to a power.

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = 6x^2 + 6x + 15$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(6x^2 + 6x + 15\right)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+1)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = 6x^2 + 6x + 15$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2 + 6x + 15 \\ \Rightarrow \frac{du}{dx} &= 12x + 6 \\ \Rightarrow du &= (12x+6)\,dx \\ \Rightarrow \frac{1}{6}du &= (2x+1)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\underbrace{\int \frac{1}{6x^2+6x+15}}_{\frac{1}{u}} \underbrace{(2x+1)\,dx}_{\frac{1}{6}du} = \int \frac{1}{u}\frac{1}{6}du = \frac{1}{6}\int \frac{1}{u}du$$

4. Integrate (in terms of u).

   $\frac{1}{6}\int \frac{1}{u}du = \frac{1}{6}\ln|u| + C$

5. Re-express in terms of the original variable, x.

   $\int \frac{2x+1}{6x^2+6x+15}dx = \underbrace{\frac{1}{6}\ln\left|6x^2+6x+15\right| + C}_{\frac{1}{6}\ln|u|+C}$

   $$\text{i.e., } \int \frac{2x+1}{6x^2+6x+15}dx = \frac{1}{6}\ln|6x^2+6x+15| + C$$

Extra! (Wow! - 5 pts. Can you believe it?) **Compute:** $\frac{d}{dx}\left[\ln\left(\sqrt{\frac{8x^4-3x^2+2}{5x^3-5x}}\right)\right] =$

$$\frac{d}{dx}\left[\ln\left(\sqrt{\frac{8x^4-3x^2+2}{5x^3-5x}}\right)\right] \underset{\text{rewrite}}{=} \frac{d}{dx}\left[\ln\left[\left(\frac{8x^4-3x^2+2}{5x^3-5x}\right)^{\frac{1}{2}}\right]\right] \underset{\text{rewrite}}{=} \frac{d}{dx}\left[\frac{1}{2}\ln\left(\frac{8x^4-3x^2+2}{5x^3-5x}\right)\right]$$

$$\underset{\text{rewrite}}{=} \frac{1}{2}\frac{d}{dx}\left[\ln\left(\frac{8x^4-3x^2+2}{5x^3-5x}\right)\right] \underset{\text{rewrite}}{=} \frac{1}{2}\frac{d}{dx}\left[\ln\left(8x^4-3x^2+2\right) - \ln\left(5x^3-5x\right)\right]$$

$$\underset{\text{rewrite}}{=} \frac{1}{2}\left(\frac{1}{8x^4-3x^2+2}\left(32x^3-6x\right) - \frac{1}{5x^3-5x}\left(15x^2-5\right)\right) \underset{\text{rewrite}}{=} \frac{16x^3-3x}{8x^4-3x^2+2} - \frac{3x^2-1}{x^3-5x}$$

$$\frac{d}{dx}\left[\ln\left(\sqrt{\frac{8x^4-3x^2+2}{5x^3-5x}}\right)\right] = \frac{16x^3-3x}{8x^4-3x^2+2} - \frac{3x^2-1}{x^3-5x}$$