MTH 1125 Test #1 - Solutions

Summer 2018

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2} \frac{x^2+5}{x^2-2} =$

i) Try Plugging in:

$\lim_{x\to 2} \frac{x^2+5}{x^2-2} = \frac{(2)^2+5}{(2)^2-2} = \frac{9}{2}$

i.e., $\lim_{x\to 2} \frac{x^2+5}{x^2-2} = \frac{9}{2}$

2. Compute: $\lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} =$

i) Try Plugging in:

$\lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} = \frac{(-1)^2+4(-1)+3}{(-1)^2-(-1)-2} = \frac{1-4+3}{1+1-2} = \frac{0}{0}$ No Good - Zero Divide!

ii) Try Factoring and Cancelling:

$\lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} = \lim_{x\to -1} \frac{(x+1)(x+3)}{(x+1)(x-2)} = \lim_{x\to -1} \frac{(x+3)}{(x-2)} = \frac{(-1)+3}{(-1)-2} = \frac{2}{-3} = -\frac{2}{3}$

i.e., $\lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} = -\frac{2}{3}$

3. Compute: $\lim_{x\to 1}\frac{x-5}{x^2-5x+4} =$

i) Try Plugging in:

$\lim_{x\to 1}\frac{x-5}{x^2-5x+4} = \frac{(1)-5}{(1)^2-5(1)+4} = \frac{-4}{0}$ No Good - Zero Divide!

ii) Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

iii) Analyze the one-sided limits:

$$\lim_{x\to 1^-}\frac{x-5}{x^2-5x+4} = \lim_{x\to 1^{--}}\frac{x-5}{(x-1)(x-4)} = \frac{-4}{(-\varepsilon)(-3)} = \frac{\left(\frac{-4}{-3}\right)}{(-\varepsilon)} = \frac{\left(\frac{4}{3}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 1^- \\ \Rightarrow & x < 1 \\ \Rightarrow & x - 1 < 0 \end{array}$$

$$\lim_{x\to 1^+}\frac{x-5}{x^2-5x+4} = \lim_{x\to 1^+}\frac{x-5}{(x-1)(x-4)} = \frac{-4}{(+\varepsilon)(-3)} = \frac{\left(\frac{-4}{-3}\right)}{(+\varepsilon)} = \frac{\left(\frac{4}{3}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 1^+ \\ \Rightarrow & x > 1 \\ \Rightarrow & x - 1 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to 1}\frac{x-5}{x^2-5x+4}$ **Does Not Exist!**

4. $f(x) = 3x^6 - 3x^5 + 4x^3 - 6x^2 + 9x - 5$; Compute: $f'(x)$.

$f'(x) = 3(6x^5) - 3(5x^4) + 4(3x^2) - 6(2x) + 9 - 0 = 18x^5 - 15x^4 + 12x^2 - 12x + 9$

i.e., $f'(x) = 18x^5 - 15x^4 + 12x^2 - 12x + 9$

5. Find the asymptotes and graph: $f(x) = \frac{x^2+2x-3}{x^2-2x-3}$

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0$

$\Rightarrow x = -1$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to -1^-} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to -1^-} \frac{x^2+2x-3}{(x+1)(x-3)} = \frac{-4}{(-\varepsilon)(-4)} = \frac{\left(\frac{-4}{-4}\right)}{(-\varepsilon)} = \frac{1}{-\varepsilon} = -\infty$$

$$\begin{array}{ll} & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x+1 < 0 \end{array}$$

$$\lim_{x\to -1^+} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to -1^+} \frac{x^2+2x-3}{(x+1)(x-3)} = \frac{-4}{(+\varepsilon)(-4)} = \frac{\left(\frac{-4}{-4}\right)}{(+\varepsilon)} = \frac{1}{+\varepsilon} = +\infty$$

$$\begin{array}{ll} & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x+1 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -1$ *IS a vertical asymptote.*

$$\lim_{x\to 3^-} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to 3^-} \frac{x^2+2x-3}{(x+1)(x-3)} = \frac{12}{(4)(-\varepsilon)} = \frac{\left(\frac{12}{4}\right)}{(-\varepsilon)} = \frac{3}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \end{array}$$

$$\lim_{x\to 3^+} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to 3^+} \frac{x^2+2x-3}{(x+1)(x-3)} = \frac{12}{(4)(+\varepsilon)} = \frac{\left(\frac{12}{4}\right)}{(+\varepsilon)} = \frac{3}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = 3$ *IS a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

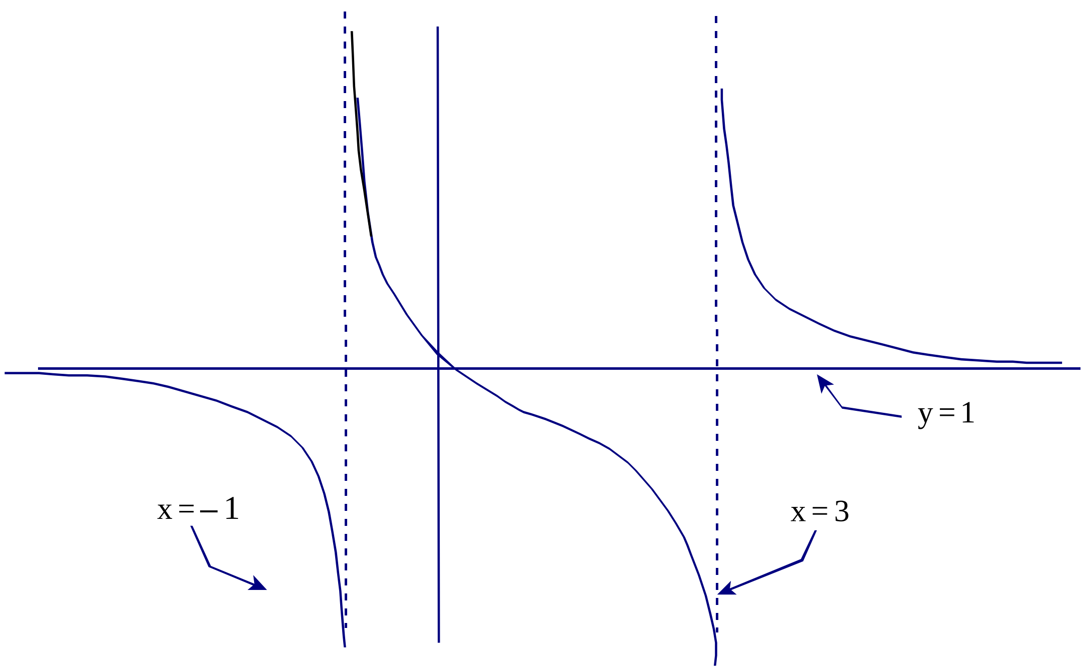
$\lim_{x\to+\infty} \frac{x^2+2x-3}{x^2-2x-3} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are finite and constant, $y = 1$ is a horizontal asymptote.

Summary:

$\lim_{x\to-1^-} \frac{x^2+2x-3}{x^2-2x-3} = -\infty$	
$\lim_{x\to-1^+} \frac{x^2+2x-3}{x^2-2x-3} = +\infty$	$\lim_{x\to-\infty} \frac{x^2+2x-3}{x^2-2x-3} = 1$
$\lim_{x\to3^-} \frac{x^2+2x-3}{x^2-2x-3} = -\infty$	$\lim_{x\to+\infty} \frac{x^2+2x-3}{x^2-2x-3} = 1$
$\lim_{x\to3^+} \frac{x^2+2x-3}{x^2-2x-3} = +\infty$	

Graph $f(x) = \frac{x^2+2x-3}{x^2-2x-3}$



6. Compute: $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} =$

i) Try Plugging in:

$\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \frac{\sqrt{(2)+2}-2}{(2)-2} = \frac{0}{0}$ No Good - Zero Divide!

ii) Try Factoring and Cancelling:

$$\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} \cdot \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2} = \lim_{x\to 2} \frac{\left(\sqrt{x+2}\right)^2-(2)^2}{(x-2)\left[\sqrt{x+2}+2\right]}$$

$$= \lim_{x\to 2} \frac{x+2-4}{(x-2)\left[\sqrt{x+2}+2\right]} = \lim_{x\to 2} \frac{(x-2)}{(x-2)\left[\sqrt{x+2}+2\right]} = \lim_{x\to 2} \frac{1}{\sqrt{x+2}+2}$$

$$= \frac{1}{\sqrt{(2)+2}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$\boxed{\text{i.e., } \lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} = \frac{1}{4}}$$

7.

$x =$	$f(x) =$
2.5	-10.1
2.9	-100.8
2.99	$-1,000.3$
2.999	$-10,000.3$
2.9999	$-100,000.9$

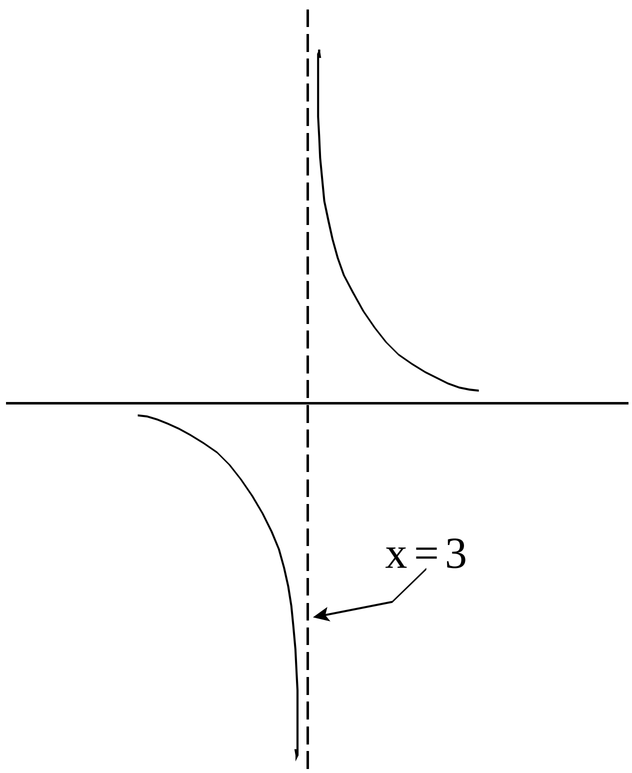
$x =$	$f(x) =$
3.5	10.1
3.1	100.8
3.01	$1,000.3$
3.001	$10,000.3$
3.0001	$100,000.9$

Based on the information in the table above, do the following:

(a) $\lim_{x\to 3^-} f(x) = -\infty$

(b) $\lim_{x\to 3^+} f(x) = +\infty$

(c) Graph $f(x)$



8. Compute: $\lim_{x\to -\infty} \frac{8x^5+5x-2x}{2x^4-5x^2-5} =$

$\lim_{x\to -\infty} \frac{8x^5+5x-2x}{2x^4-5x^2-5} = \lim_{x\to -\infty} \frac{8x^5}{2x^4} = \lim_{x\to -\infty} 4x = -\infty$

i.e., $\lim_{x\to -\infty} \frac{8x^5+5x-2x}{2x^4-5x^2-5} = -\infty$

9. Compute: $\frac{d}{dx}\left[4\sin(x) + 12\cos(x)\right] =$

$\frac{d}{dx}\left[4\sin(x) + 12\cos(x)\right] = 4\left(\cos(x)\right) + 12\left(-\sin(x)\right) = 4\cos(x) - 12\sin(x)$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[4\sin(x) + 12\cos(x)\right] = 4\cos(x) - 12\sin(x)}$$

10. $f(x) = 6x^2 - 2x$; compute $f'(x)$ **using the definition of derivative.** (i.e., compute $f'(x)$ using the "limiting process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[6(x+\Delta x)^2 - 2(x+\Delta x)\right] - \left(6x^2 - 2x\right)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[6\left(x^2 + 2x\Delta x + \Delta x^2\right) - 2(x+\Delta x)\right] - \left(6x^2 - 2x\right)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[\left(6x^2 + 12x\Delta x + 6\Delta x^2\right) - (2x + 2\Delta x)\right] - \left(6x^2 - 2x\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{12x\Delta x + 6\Delta x^2 - 2\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(12x + 6\Delta x - 2)}{\Delta x} = \lim_{\Delta x \to 0} (12x + 6\Delta x - 2) = 12x + 6(0) - 2 = 12x - 2$

$$\boxed{\text{i.e., } f'(x) = 12x - 2}$$

11. Compute: $\frac{d}{dx}\left[\left(x^3 + 2x\right)\sin(x)\right] =$

$$\frac{d}{dx}\left[\underbrace{\left(x^3 + 2x\right)}_{1^{\text{st}}} \cdot \underbrace{\sin(x)}_{2^{\text{nd}}}\right] = \underbrace{\left(3x^2 + 2\right)}_{1^{\text{st}} \text{ prime}} \cdot \underbrace{\sin(x)}_{2^{\text{nd}}} + \underbrace{\cos(x)}_{2^{\text{nd}} \text{prime}} \cdot \underbrace{\left(x^3 + 2x\right)}_{1^{\text{st}}}$$

$$= \left(3x^2 + 2\right)\sin(x) + \left(x^3 + 2x\right)\cos(x)$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\left(x^3 + 2x\right)\sin(x)\right] = \left(3x^2 + 2\right)\sin(x) + \left(x^3 + 2x\right)\cos(x)}$$

12. Compute: $\frac{d}{dx}\left[\frac{\cos(x)}{4x^3 + 2x^2}\right] =$

$$\frac{d}{dx}\left[\frac{\overbrace{\cos(x)}^{\text{top}}}{\underbrace{4x^3 + 2x^2}_{\text{bottom}}}\right] = \frac{\overbrace{\left(-\sin(x)\right)}^{\text{top prime}} \cdot \overbrace{\left(4x^3 + 2x^2\right)}^{\text{bottom}} - \overbrace{\left(12x^2 + 4x\right)}^{\text{bottom prime}} \cdot \overbrace{\cos(x)}^{\text{top}}}{\underbrace{\left(4x^3 + 2x^2\right)^2}_{\text{bottom squared}}}$$

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\frac{\cos(x)}{4x^3 + 2x^2}\right] = -\frac{\sin(x)\left(4x^3 + 2x^2\right) + \left(12x^2 + 4x\right)\cos(x)}{\left(4x^3 + 2x^2\right)^2}}$$