MTH 1125 Test #2 - Solutions

Summer 2020

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Show CLEARLY how you arrive at your answers

1. Compute: $\frac{d}{dx} = \left[5x^4 + 6x^3 + 10x^2 + 18x + 5 + 10\sqrt{x}\right] =$

   $\frac{d}{dx}\left[5x^4 + 6x^3 + 10x^2 + 18x + 5 + 10\sqrt{x}\right]$

   $= 5\left[4x^3\right] + 6\left[3x^2\right] + 10\left[2x\right] + 18 + 0 + 10\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

   $= 20x^3 + 18x^2 + 20x + 18 + 5x^{-\frac{1}{2}}$

$$\text{i.e., } \frac{d}{dx}\left[5x^4 + 6x^3 + 10x^2 + 18x + 5 + 10\sqrt{x}\right] = 20x^3 + 18x^2 + 20x + 18 + 5x^{-\frac{1}{2}}$$

2. Compute: $\frac{d}{dx}\left[\left(4x^3 + 6x^2\right)\left(8x^5 + 5x^4 - 6\right)\right] =$

$$\frac{d}{dx}\left[\underbrace{\left(4x^3 + 6x^2\right)}_{1^{st}}\underbrace{\left(8x^5 + 5x^4 - 6\right)}_{2^{nd}}\right] = \underbrace{\left(12x^2 + 12x\right)}_{1^{st}\text{ prime}} \cdot \underbrace{\left(8x^5 + 5x^4 - 6\right)}_{2^{nd}} + \underbrace{\left(40x^4 + 20x^3\right)}_{2^{nd}\text{ prime}} \cdot \underbrace{\left(4x^3 + 6x^2\right)}_{1^{st}}$$

$$\frac{d}{dx}\left[\left(4x^3 + 6x^2\right)\left(8x^5 + 5x^4 - 6\right)\right] = \left(12x^2 + 12x\right)\left(8x^5 + 5x^4 - 6\right) + \left(40x^4 + 20x^3\right)\left(4x^3 + 6x^2\right)$$

3. Compute: $\frac{d}{dx}\left[\frac{\sin(x)}{7x^2 + 8x + 6}\right] =$

$$\frac{d}{dx}\left[\frac{\overbrace{\sin(x)}^{\text{top}}}{\underbrace{7x^2 + 8x + 6}_{\text{Bottom}}}\right] = \frac{\overbrace{\cos(x)}^{\text{top prime}} \cdot \overbrace{\left(7x^2 + 8x + 6\right)}^{\text{bottom}} - \overbrace{(14x + 8)}^{\text{bottom prime}} \cdot \overbrace{\sin(x)}^{\text{top}}}{\underbrace{\left(7x^2 + 8x + 6\right)^2}_{\substack{\text{bottom}\\\text{squared}}}}$$

$$\text{i.e., } \frac{d}{dx}\left[\frac{\sin(x)}{7x^2 + 8x + 6}\right] = \frac{\cos(x)\left(7x^2 + 8x + 6\right) - (14x + 8)\sin(x)}{\left(7x^2 + 8x + 6\right)^2}$$

4. Compute: $\frac{d}{dx}\left[\left(5x^3+3x^2+7x+8\right)^{10}\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(5x^3+3x^2+7x+8\right)^{10}\right] = \underbrace{10\left(5x^3+3x^2+7x+8\right)^9}_{\text{power rule as usual}} \cdot \underbrace{\left(15x^2+6x+7\right)}_{\text{derivative of inner}}$$

i.e., $\frac{d}{dx}\left[\left(5x^3+3x^2+7x+8\right)^{10}\right] = 10\left(5x^3+3x^2+7x+8\right)^9\left(15x^2+6x+7\right)$

5. Compute: $\frac{d}{dx}\left[\sec\left(6x^4+12x^2+24x\right)\right] =$

Outer:	$=$	$\sec(\quad)$
Deriv. of outer	$=$	$\sec(\quad)\tan(\quad)$

$$\frac{d}{dx}\left[\underset{\substack{\uparrow\\ \text{outer}}}{\sec}\left(\underbrace{6x^4+12x^2+24x}_{\substack{\uparrow\\ \text{inner}}}\right)\right] = \underbrace{\sec\left(6x^4+12x^2+24x\right)\tan\left(6x^4+12x^2+24x\right)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\left(24x^3+24x+24\right)}_{\text{deriv. of inner}}$$

i.e., $\frac{d}{dx}\left[\sec\left(6x^4+12x^2+24x\right)\right] = \sec\left(6x^4+12x^2+24x\right)\tan\left(6x^4+12x^2+24x\right)\left(24x^3+24x+24\right)$

6. Given that $y = \cot(x)$ and $x = 3t^2 + 3t + 3$, compute $\frac{dy}{dt}$ using the Liebniz form of the chain rule.

We know:

$\frac{dy}{dx} = -\csc^2(x)$

$\frac{dx}{dt} = 6t + 3$

We want: $\frac{dy}{dt}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = -\csc^2(x)(6t+3) = \underbrace{-\csc^2\left(3t^2+3t+3\right)(6t+3)}_{\substack{\text{express solely in terms of} \\ \text{independent variable } v}}$$

i.e. $\frac{dy}{dt} = -\csc^2(3t^2+3t+3)(6t+3)$

7. Given that $f(x) = 6x^2 + 3x - 5$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[6(x+\Delta x)^2 + 3(x+\Delta x) - 5\right] - \left[6x^2+3x-5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[6\left(x^2+2x\Delta x+\Delta x^2\right)+3(x+\Delta x)-5\right] - \left[6x^2+3x-5\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[6x^2+12x\Delta x+6\Delta x^2+3x+3\Delta x-5\right] - \left[6x^2+3x-5\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{12x\Delta x+6\Delta x^2+3\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(12x+6\Delta x+3)}{\Delta x} = \lim_{\Delta x \to 0}(12x+6\Delta x+3) = 12x + 6(0) + 3 = 12x + 3$$

i.e., $f'(x) = 12x + 3$

8. Given that $x^3 + 6x^3y^5 = \sin(y) + \cos(x)$, compute y'

i. Differentiate both sides w.r.t. x

$$\frac{d}{dx}\left[x^3 + \underbrace{6x^3}_{1^{\text{st}}}\underbrace{y^5}_{2^{\text{nd}}}\right] = \frac{d}{dx}\left[\sin(y) + \cos(x)\right]$$

$$\Rightarrow 3x^2 + \left(\underbrace{18x^2}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^5}_{2^{\text{nd}}} + \underbrace{5y^4y'}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{6x^3}_{1^{\text{st}}}\right) = \cos(y) \cdot y' - \sin(x)$$

Simplifying, we have:

$$3x^2 + 18x^2y^5 + 30x^3y^4y' = \cos(y) \cdot y' - \sin(x)$$

ii. Solve algebraically for y'

a. Get y' terms on left side, all other terms on right side

$$\Rightarrow 30x^3y^4y' - \cos(y)\,y' = -\sin(x) - 3x^2 - 18x^2y^5$$

b. Factor out y'

$$\Rightarrow \left(30x^3y^4 - \cos(y)\right)y' = -\sin(x) - 3x^2 - 18x^2y^5$$

c. Divide both sides by the cofactor of y'

$$y' = \frac{-\sin(x) - 3x^2 - 18x^2y^5}{30x^3y^4 - \cos(y)} = \frac{\sin(x) + 3x^2 + 18x^2y^5}{\cos(y) - 30x^3y^4}$$

$$\boxed{y' = \frac{-\sin(x) - 3x^2 - 18x^2y^5}{30x^3y^4 - \cos(y)} = \frac{\sin(x) + 3x^2 + 18x^2y^5}{\cos(y) - 30x^3y^4}}$$

9. Compute: $\frac{d}{dx}\left[\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^5\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^5}_{(g(x))^n}\right] = \underbrace{5\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{5x^4+10x^2+3}{8x^3+12x^2}\right]\right)}_{\substack{\text{deriv of}\\ \text{inner Function}}}$$

$$= 5\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^4 \underbrace{\frac{(20x^3+20x)(8x^3+12x^2) - (24x^2+24x)(5x^4+10x^2+3)}{(8x^3+12x^2)^2}}_{\substack{\text{quotient}\\ \text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^5\right] = 5\left(\frac{5x^4+10x^2+3}{8x^3+12x^2}\right)^4 \frac{\left(20x^3+20x\right)\left(8x^3+12x^2\right) - \left(24x^2+24x\right)\left(5x^4+10x^2+3\right)}{(8x^3+12x^2)^2}$

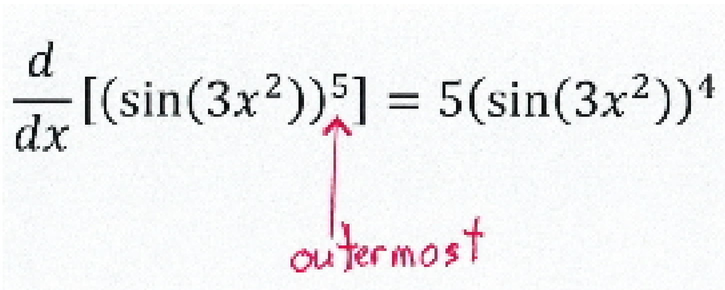
10. Compute: $\frac{d}{dx}\left[\sin^5\left(3x^2\right)\right] =$

Let's rewrite this:

$\frac{d}{dx}\left[\left(\sin\left(3x^2\right)\right)^5\right]$

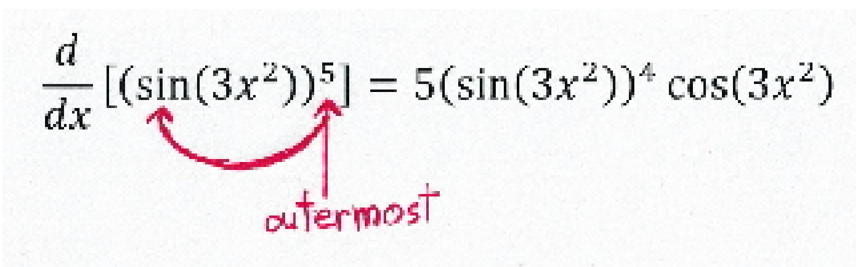
This is the composition of *three* functions.

Differentiate the outermost function and evaluate it at everything inside



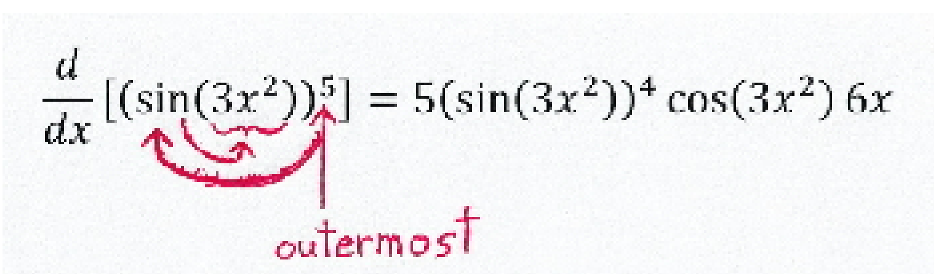
This yields: $5\left(\sin\left(3x^2\right)\right)^4$

Next: Multiply by the derivative of the next outermost function and evaluate it at everything inside of it.



This yields: $5\left(\sin\left(3x^2\right)\right)^4 \cdot \cos\left(3x^2\right)$

Finally: Multiply by the derivative of the innermost function.



This yields: $5\left(\sin\left(3x^2\right)\right)^4 \cdot \cos\left(3x^2\right) \cdot (6x)$

i.e., $\frac{d}{dx}\left[\sin^5\left(3x^2\right)\right] = 5\left(\sin\left(3x^2\right)\right)^4 \cdot \cos\left(3x^2\right) \cdot (6x)$