MTH 1112 - Test #1 - Solutions

Summer 2021

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Show CLEARLY how you arrive at your answers.

1. Solve the equation: $\frac{x}{x+5} = \frac{7}{6}$

$\frac{x}{x+n} = \frac{7}{6}$

$\Rightarrow \frac{x}{x+5}(x+5) = \frac{7}{6}(x+5) \quad x \neq -5$

$\Rightarrow x = \frac{7}{6}(x+5)$

$\Rightarrow nx + x = nx + n^2 + 2x + 2n$

$\Rightarrow 6x = (6)\frac{7}{6}(x+5)$

$\Rightarrow 6x = 7(x+5)$

$\Rightarrow 6x = 7x + 35$

$\Rightarrow -x = 35$

$\Rightarrow x = -35$

$\boxed{x = x = -35}$

2. Solve the equation: $x(4x-4) = (4x+8)(x-6)$

$x(4x-4) = (4x+8)(x-6) : 4x^2 - 16x - 48$

$\Rightarrow 4x^2 - 4x = 4x^2 + 8x - 24x - 48$

$\Rightarrow 4x^2 - 4x = 4x^2 - 16x - 48$

$\Rightarrow -4x = -16x - 48$

$\Rightarrow 12x = -48$

$\Rightarrow x = -4$

$\boxed{x = -4}$

3. Solve by factoring: $16x^2 - 40x + 25 = 0$

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$\Rightarrow (4x-5)(4x-5) = 0$

$\Rightarrow (4x-5)^2 = 0$

$\Rightarrow 4x - 5 = 0$

$\Rightarrow 4x = 5$

$\Rightarrow x = \frac{5}{4}$

$\boxed{x = \frac{5}{4}}$

4. Solve the equation by the square root method: $(3x+9)^2 = 81$

$(3x+9)^2 = 81$

$\Rightarrow \sqrt{(3x+9)^2} = \pm\sqrt{81}$

$\Rightarrow 3x + 9 = \pm 9$

$\Rightarrow 3x = -9 \pm 9$

$\Rightarrow 3x = -9 + 9$ or $3x = -9 - 9$

$\Rightarrow 3x = 0$ or $3x = -18$

$\Rightarrow x = 0$ or $x = -6$

$\boxed{x = 0 \text{ or } x = -6}$

5. Solve the equation using the Quadratic Formula: $4x^2 - 8x + 2 = 0$

$$\underbrace{4}_{a}x^2 + \underbrace{(-8)}_{b}x + \underbrace{2}_{c} = 0$$

By the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(2)}}{2(4)} = \frac{8 \pm \sqrt{64 - 32}}{8} = \frac{8 \pm \sqrt{32}}{8} = \frac{8 \pm \sqrt{16 \cdot 2}}{8} = \frac{8 \pm \sqrt{16}\sqrt{2}}{8} = \frac{8 \pm 4\sqrt{2}}{8}$$

$$= 1 \pm \tfrac{1}{2}\sqrt{2}$$

i.e., $x = 1 \pm \frac{1}{2}\sqrt{2}$

$$\boxed{x = 1 - \tfrac{1}{2}\sqrt{2},\ \ 1 + \tfrac{1}{2}\sqrt{2}}$$

6. Solve the equation by Completing the Square: $4x^2 - 4x - 3 = 0$

1. Move the constant term to the other side

$\Rightarrow 4x^2 - 4x = 3$

2. Divide both sides by the coefficient of x^2

$\Rightarrow x^2 - x = \frac{3}{4}$

3. Take $\frac{1}{2}$ of the coefficient of x, square it, add to both sides

$\Rightarrow x^2 - x + \left(-\frac{1}{2}\right) = \frac{3}{4} + \left(-\frac{1}{2}\right)^2$

$\Rightarrow x^2 - x + \left(-\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4}$

$\Rightarrow x^2 - x + \left(-\frac{1}{2}\right)^2 = 1$

4. Rewrite the left hand side as a perfect square

$\Rightarrow \left(x - \frac{1}{2}\right)^2 = 1$

5. Take the square root of both sides

$\Rightarrow \left(x - \frac{1}{2}\right) = \pm\sqrt{1}$

$\Rightarrow x = \frac{1}{2} \pm 1$

$\Rightarrow x = \frac{1}{2} + 1$ or $x = \frac{1}{2} - 1$

$\Rightarrow x = \frac{3}{2}$ or $x = -\frac{1}{2}$

$x = \frac{3}{2}$ or $x = -\frac{1}{2}$

7. Write the expression in standard form $a + bi$: $(2 + i)(4 - 3i)$

$(2 + i)(4 - 3i) = 8 - 6i + 4i - 3i^2 = 8 - 6i + 4i - 3(-1) = 8 - 6i + 4i + 3 = 11 - 2i$

i.e. $(2 + i)(4 - 3i) = 11 - 2i$

i.e. $(2 + i)(4 - 3i) = 11 - 2i$

8. Write the inequality using interval notation and illustrate the inequality using the real number line: $4 < x \leq 8$

Interval Notation: $(4, 8]$

Using the real number line:

9. Solve the inequality. Express your answer using set notation or interval notation. Graph the solution set. $4 - 2x \leq -10$

$4 - 2x \leq -10$

$\Rightarrow 14 - 2x \leq 0$

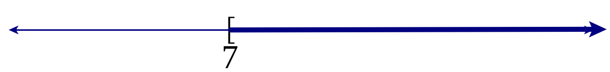
$\Rightarrow 14 \leq 2x$

$\Rightarrow 7 \leq x$

Set Notation: $\{x : 7 \leq x\}$

Interval Notation: $[7, \infty)$

Graph of Solution Set:



10. Find the distance between the points P_1 and P_2, if $P_1 = (2, 5)$ and $P_2 = (8, -3)$

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$$D = \sqrt{(-3 - 5)^2 + (8 - 2)^2} = \sqrt{(-8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

$$\boxed{D = 100}$$

11. The graph of an equation is given.

(a) Find the intercepts.

The graph crosses the y-axis at the point 3 on the y-axis.

y-intercept $= 3$

The graph crosses the x-axis at the points -4 and 4 on the x-axis.

x-intercepts $= -4$, 4

(b) Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, the origin, or none of these.

"By inspection," the portion of the graph that lies below the x-axis is clearly NOT the "mirror image" of the portion of the graph that lies above the x-axis. The graph is **NOT symmetric with respect to thex-axis.**

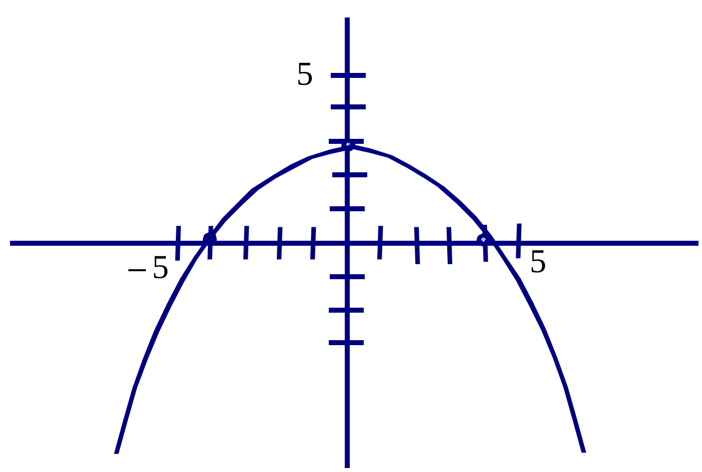
"By inspection," the portion of the graph that lies to the left of the y-axis appears to be the "mirror image" of the portion of the graph that lies to the right of the y-axis.

By definition, a graph is symmetric about the y-axis exactly when $(-x, y)$ is a point on the graph whenever (x, y) is a point on the graph. We do a "test case" to see whether this is true. Note that $(4, 0)$ is a point on the graph and $(-4, 0)$ is also a point on the graph.

We conclude that the graph **IS symmetric with respect to the x-axis.**

By definition, a graph is symmetric with respect to theorigin exactly when $(-x, -y)$ is a point on the graph whenever (x, y) is a point on the graph.

Notice that when $x = 4$, $y \approx 1$. However, when $x = -4$, $y \not\approx -1$. Thus, $(-x, -y)$ is NOT a point on the graph whenever (x, y) is a point on the graph. The graph is **NOT symmetric with respect to theorigin.**



12. For the givenequation, list the intercepts and test for symmetry. $x^2 + y - 36 = 0$

The x-intercepts of an equation are exactly those points on the graph whose y-coordinates are zero. To find these points, we set $y = 0$ and solve for x.

$\Rightarrow x^2 + (0) - 36 = 0$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

x-intercepts are $x = -6$ and $x = 6$

The y-intercepts of an equation are exactly those points on the graph whose x-coordinates are zero. To find these points, we set $x = 0$ and solve for y.

$(0)^2 + y - 36 = 0$

$\Rightarrow y - 36 = 0$

$\Rightarrow y = 36$

y-intercept is $y = 36$

Symmetry about the y-axis:

For this part, let's rewrite the equation $x^2 + y - 36 = 0$ as $y = 36 - x^2$

Observe that $36 - (-x)^2 = 36 - x^2$

Thus, if $y = 36 - x^2$, then $y = 36 - (-x)^2$

(i.e., whenever (x, y) is a point on the graph, $(-x, y)$ is also a point on the graph.)

The graph **IS symmetric about the y-axis**

Symmetry about the x-axis:

Observe that if $(x, y) = (0, 36)$, then $x^2 + y - 36 = 0$. (i.e., $(x, y) = (0, 36)$ is a point on the graph of $x^2 + y - 36 = 0$.

Now observe that if $(x, -y) = (0, -36)$, then $x^2 + y - 36 = (0)^2 + (-36) - 36 = -72$

i.e., $(x, -y) = (0, -36)$ is NOT a point on the graph of $x^2 + y - 36 = 0$

Therefore, if (x, y) is a point on the graph of $x^2 + y - 36 = 0$, the point $(x, -y)$ is NOT necessarily a point on the graph.

The graph is **NOT symmetric about the x-axis**

Symmetry about the origin:

Observe that if $(x, y) = (0, 36)$, then $x^2 + y - 36 = 0$. (i.e., $(x, y) = (0, 36)$ is a point on the graph of $x^2 + y - 36 = 0$.

Now observe that if $(-x, -y) = (-0, -36)$, then $x^2 + y - 36 = (-0)^2 + (-36) - 36 = -72$

i.e., $(-x, -y) = (0, -36)$ is NOT a point on the graph of $x^2 + y - 36 = 0$

Therefore, if (x, y) is a point on the graph of $x^2 + y - 36 = 0$, the point $(-x, -y)$ is NOT necessarily a point on the graph.

The graph is **NOT symmetric about the origin.**

13. A point on a line and its slope are given. Find thepoint-slope form of the equation of the line.

    Point: $(3, 6)$ Slope: 2

    Thepoint-slope form of the equation of a line is $(y - y_1) = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a known point on the line.

    Thus, we have: $(x_1, y_1) = (3, 6)$ and $m = 2$

    This yields: $(y - 6) = 2(x - 3)$

    $$\boxed{(y - 6) = 2(x - 3)}$$

14. The slope m and a point P on a line are given. Use the information to find two additional points on the line.

Point: $(3, 6)$ Slope: 2

Slope $m = 2 = \frac{2}{1} = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are points on the line.

We'll let $(x_1, y_1) = (3, 6)$.

Therefore, for any other point (x_2, y_2) on the line, we have: $2 = \frac{y_2 - 6}{x_2 - 3}$ or $\frac{y_2 - 6}{x_2 - 3} = 2$

$\Rightarrow y_2 - 6 = 2(x_2 - 3)$

$\Rightarrow y_2 - 6 = 2x_2 - 6$

$\Rightarrow y_2 = 2x_2$

Arbitrarily, we choose x-values to plug in to the equation.

Letting $x_2 = 1$, we have $y_2 = 2(1) = 2$

i.e., $(x_2, y_2) = (1, 2)$ is a point on the line.

Letting $x_2 = 2$, we have $y_2 = 2(2) = 4$

i.e., $(x_2, y_2) = (2, 4)$ is a point on the line.

$(1, 2)$ and $(2, 4)$ are possibilities. Other answers are possible.

15. Find the slope and y-intercept of the line. Graph the line. $\frac{1}{3}y = x + 2$

The easiest way to do this is to rewrite the equation in slope-intercept form: $y = mx+b$, where m = slope, and b = y-intercept

$\frac{1}{3}y = x + 2$

$\Rightarrow 3\left(\frac{1}{3}y\right) = 3(x+2)$

$\Rightarrow y = 3x + 6$

$\Rightarrow y = \underbrace{3}_{m} x + \underbrace{6}_{b}$

Slope: $m = 3$, y-intercept $= 6$

To graph the line, let's find one other point on the line in addition to the y-intercept.

When $x = 1$, $y = 3(1) + 6 = 9$

i.e., $(x, y) = (1, 9)$ is a point on the line

Alternatively, we can find the x-intercept. (Let $y = 0$, and solve for x)

Using the original equation: $\frac{1}{3}y = x + 2$, and letting $y = 0$, we have:

$\frac{1}{3}(0) = x + 2 \Rightarrow 0 = x + 2 \Rightarrow x = -2$ (x-intercept)