MTH 1125 (1 pm) Test #1 - Solutions

FALL 2010

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Instructions. Show CLEARLY how you arrive at your answers

1. Compute: $\lim_{x\to 3} \frac{x^2-2x-3}{x^2+2x-15} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 3} \frac{x^2-2x-3}{x^2+2x-15} = \frac{(3)^2-2(3)-3}{(3)^2+2(3)-15} = \frac{0}{0}$ No Good - Division by Zero

   2. Try factoring and canceling

   $\lim_{x\to 3} \frac{x^2-2x-3}{x^2+2x-15} = \lim_{x\to 3} \frac{(x+1)(x-3)}{(x+5)(x-3)} = \lim_{x\to 3} \frac{(x+1)}{(x+5)} = \frac{(3)+1}{(3)+5} = \frac{1}{2}$

   i.e., $\lim_{x\to 3} \frac{x^2-2x-3}{x^2+2x-15} = \frac{1}{2}$

2. Compute: $\lim_{x\to 2} \frac{2x+5}{x^2+5} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 2} \frac{2x+5}{x^2+5} = \frac{2(2)+5}{(2)^2+5} = 1$

   i.e., $\lim_{x\to 2} \frac{2x+5}{x^2+5} = 1$

3. Compute: $\lim_{x\to -1} \frac{x+5}{x^2-2x-3} =$

(a) 1. Try plugging in:

$\lim_{x\to -1} \frac{x+5}{x^2-2x-3} = \frac{(-1)+5}{(-1)^2-2(-1)-3} = \frac{4}{0}$ No Good - Division by Zero

2. Try factoring and canceling

No Good - factoring and canceling only works when Step #1 yields $\frac{0}{0}$.

3. Analyze one-sided limits

$\lim_{x\to -1^-} \frac{x+5}{x^2-2x-3} = \lim_{x\to -1^-} \frac{x+5}{(x+1)(x-3)} = \frac{4}{(-\varepsilon)(-4)} = \frac{4}{(\varepsilon)(4)} = \frac{1}{(\varepsilon)} = +\infty$

$$\begin{array}{ll} & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x+1 < 0 \end{array}$$

$\lim_{x\to -1^+} \frac{x+5}{x^2-2x-3} = \lim_{x\to -1^+} \frac{x+5}{(x+1)(x-3)} = \frac{4}{(+\varepsilon)(-4)} = \frac{\left(\frac{4}{-4}\right)}{(\varepsilon)} = \frac{-1}{(\varepsilon)} = -\infty$

$$\begin{array}{ll} & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x+1 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to -1} \frac{x+5}{x^2-2x-3}$ Does Not Exist.

4. Compute: $\lim_{x\to 8} \frac{\sqrt{x+8}-4}{x-8} =$

(a) 1. Try plugging in:

$\lim_{x\to 8} \frac{\sqrt{x+8}-4}{x-8} = \frac{\sqrt{(8)+8}-4}{(8)-8} = 0$ No Good - Division by Zero

2. Try factoring and canceling

$\lim_{x\to 8} \frac{\sqrt{x+8}-4}{x-8} = \lim_{x\to 8} \frac{\sqrt{x+8}-4}{x-8} \cdot \frac{\sqrt{x+8}+4}{\sqrt{x+8}+4} = \lim_{x\to 8} \frac{\left(\sqrt{x+8}\right)^2-(4)^2}{(x-8)\left[\sqrt{x+8}+4\right]} =$

$\lim_{x\to 8} \frac{(x+8)-16}{(x-8)\left[\sqrt{x+8}+4\right]} = \lim_{x\to 8} \frac{x-8}{(x-8)\left[\sqrt{x+8}+4\right]} = \lim_{x\to 8} \frac{1}{\left[\sqrt{x+8}+4\right]} =$

$\frac{1}{\left[\sqrt{(8)+8}+4\right]} = \frac{1}{4+4} = \frac{1}{8}$

i.e., $\lim_{x\to 8} \frac{\sqrt{x+8}-4}{x-8} = \frac{1}{8}$

5. Compute: $\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^2+5x+5} =$

$\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^2+5x+5} = \lim_{x\to\infty} \frac{5x^3}{5x^2} = \lim_{x\to\infty} x = +\infty$

i.e., $\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^2+5x+5} = +\infty$

6.

$x =$	$f(x) =$
1.5	−15.1
1.9	−227.8
1.99	−1212.3
1.999	−21156.3
1.9999	−834561.9

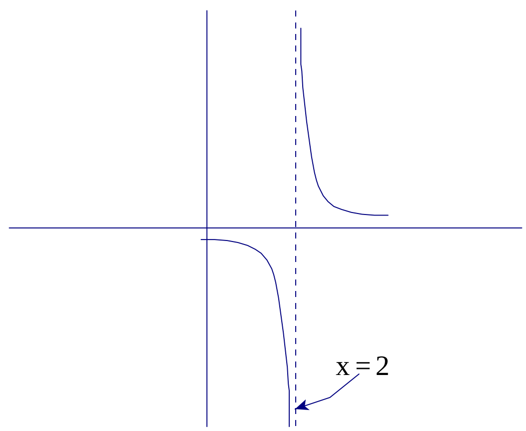
$x =$	$f(x) =$
2.5	15.1
2.1	227.8
2.01	1212.3
2.001	21156.3
2.0001	834561.9

Based on the information in the table above, do the following:

(a) $\lim_{x\to 2^-} f(x) = -\infty$

(b) $\lim_{x\to 2^+} f(x) = +\infty$

(c) Graph $f(x)$



7. Determine whether $f(x)$ is continuous at the point $x = 3$.

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{for } x < 3 \\ 2x - 3 & \text{for } x \geq 3 \end{cases}$$

$f(x)$ is continuous at the point $x = 3$ exactly when $\lim_{x \to 3} f(x) = f(3)$.

We will check this by computing $\lim_{x \to 3} f(x)$.

Since $f(x)$ is defined differently for $x < 3$ than it is for $x > 3$, we must compute the one-sided limits in order to compute $\lim_{x \to 3} f(x)$.

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \lim_{x \to 3^-} \frac{(x+3)(x-3)}{(x-3)} = \lim_{x \to 3^-} (x+3) = ((3)+3) = 6$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2x-3) = (2(3)-3) = 3$

Since the one-sided limits are not equal, $\lim_{x \to 3} f(x)$ does not exist.

Since $\lim_{x \to 3} f(x)$ does not exist, $\lim_{x \to 3} f(x) \neq f(3)$.

Since $\lim_{x \to 3} f(x) \neq f(3)$, $f(x)$ is NOT continuous at the point, $x = 3$.

8. $f(x) = \frac{x^3}{x^2-1}$. Find the asymptotes and graph

Verticals

Find x-values that cause division by zero.

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x+1)(x-1) = 0$

$\Rightarrow x = -1$ and $x = 1$ *possible* vertical asymptotes

Compute the one-sided limits

$x = -1$

$\lim_{x\to -1^-} \frac{x^3}{x^2-1} = \lim_{x\to -1^-} \frac{x^3}{(x+1)(x-1)} = \frac{-1}{(-\varepsilon)(-2)} = \frac{-1}{(\varepsilon)(2)} = \frac{\left(-\frac{1}{2}\right)}{\varepsilon} = -\infty$

$$\begin{array}{|ll|}\hline & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x + 1 < 0 \\ \hline\end{array}$$

Infinite limits tell us that $x = -1$ IS a vertical asymptote

$\lim_{x\to -1^+} \frac{x^3}{x^2-1} = \lim_{x\to -1^+} \frac{x^3}{(x+1)(x-1)} = \frac{-1}{(\varepsilon)(-2)} = \frac{1}{(\varepsilon)(2)} = \frac{\left(\frac{1}{2}\right)}{\varepsilon} = +\infty$

$$\begin{array}{|ll|}\hline & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x + 1 > 0 \\ \hline\end{array}$$

$x = 1$

$\lim_{x\to 1^-} \frac{x^3}{x^2-1} = \lim_{x\to 1^-} \frac{x^3}{(x+1)(x-1)} = \frac{1}{(2)(-\varepsilon)} = \frac{\left(-\frac{1}{2}\right)}{\varepsilon} = -\infty$

$$\begin{array}{|ll|}\hline & x \to 1^- \\ \Rightarrow & x < 1 \\ \Rightarrow & x - 1 < 0 \\ \hline\end{array}$$

Infinite limits tell us that $x = 1$ IS a vertical asymptote

$\lim_{x\to 1^+} \frac{x^3}{x^2-1} = \lim_{x\to 1^+} \frac{x^3}{(x+1)(x-1)} = \frac{1}{(2)(\varepsilon)} = \frac{\left(\frac{1}{2}\right)}{\varepsilon} = +\infty$

$$\begin{array}{|ll|}\hline & x \to 1^+ \\ \Rightarrow & x > 1 \\ \Rightarrow & x - 1 > 0 \\ \hline\end{array}$$

Horizontals

Compute limits as $x \to -\infty$ and $x \to +\infty$.

$\lim_{x\to-\infty} \frac{x^3}{x^2-1} = \lim_{x\to-\infty} \frac{x^3}{x^2} = \lim_{x\to-\infty} x = -\infty$

Limits are not *finite* and *constant*. Hence, there is NO horizontal asymptote

$\lim_{x\to+\infty} \frac{x^3}{x^2-1} = \lim_{x\to+\infty} \frac{x^3}{x^2} = \lim_{x\to+\infty} x = +\infty$

Graph $f(x) = \frac{x^3}{x^2-1}$