Type b Critical Numbers - Solutions

Fall 2015

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In each exercise, determine the intervals on which $f(x)$ is increasing/decreasing, and identify all relative maximums and minimums.

1. $f(x) = \frac{16}{5}x^{\frac{5}{3}} + x^{\frac{2}{3}} + 2$

   i. Compute $f'(x)$ and find critical numbers

   $f'(x) = \frac{16}{3}x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}} = \frac{16x^{\frac{2}{3}}}{3} + \frac{2}{3x^{\frac{1}{3}}} = \frac{16x^{\frac{2}{3}}}{3}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} + \frac{2}{3x^{\frac{1}{3}}} = \frac{16x^{\frac{3}{3}}}{3} + \frac{2}{3x^{\frac{1}{3}}} = \frac{16x+2}{3x^{\frac{1}{3}}}$

   i.e., $f'(x) = \frac{16x+2}{3x^{\frac{1}{3}}}$

   a. "Type a" $(f'(c) = 0)$

   Set $f'(x) = \frac{16x+2}{3x^{\frac{1}{3}}} = 0$

   $\Rightarrow 16x + 2 = 0$

   $\Rightarrow 16x = -2$
   $\Rightarrow x = -\frac{1}{8}$ critical number

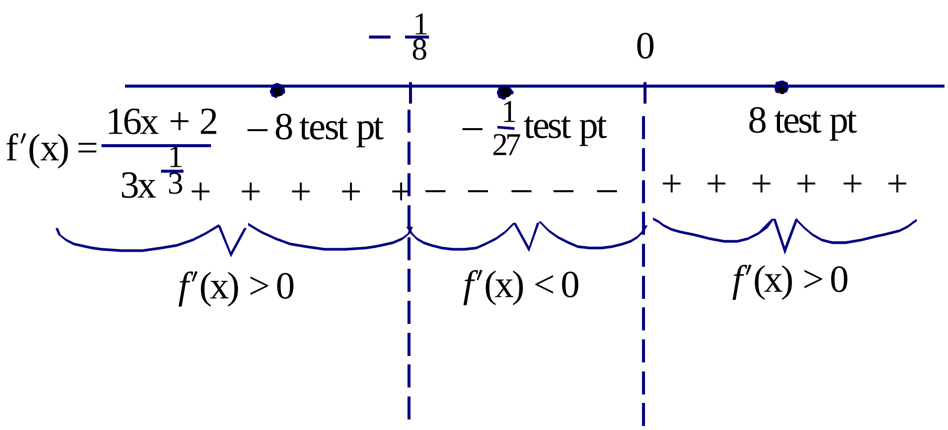
   b. "Type b" $(f'(c)$ undefined$)$

   Set denominator $3x^{\frac{1}{3}} = 0$

   $\Rightarrow x = 0$ critical number

   ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

   iii. From each interval select a "test point" to plug into $f'(x)$



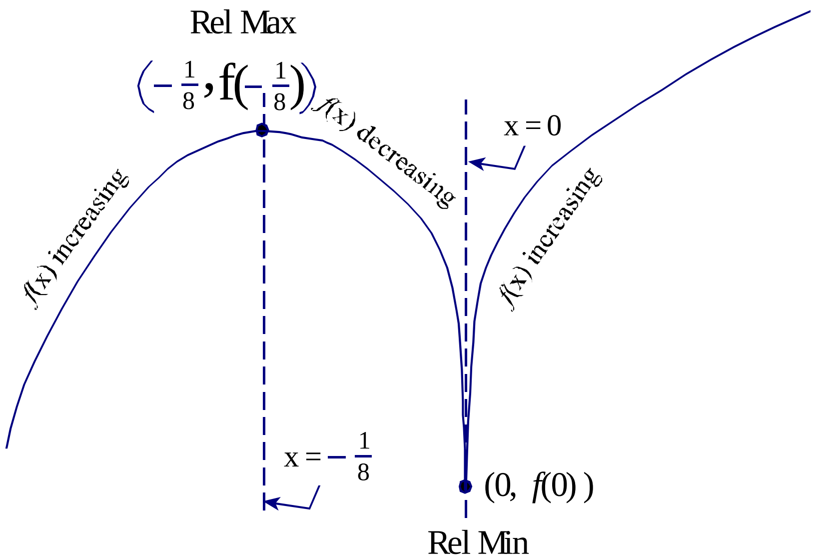
   $f(x)$ is **increasing** on the intervals $\left(-\infty, -\frac{1}{8}\right)$ and $(0, \infty)$

   (Because $f'(x)$ is positive on these intervals)

   $f(x)$ is **decreasing** on the intervals and $\left(-\frac{1}{8}, 0\right)$

   (Because $f'(x)$ is negative on this interval)

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals and $\left(-\infty, -\frac{1}{8}\right)$ and $(0, \infty)$

$f(x)$ is **decreasing** on the interval $\left(-\frac{1}{8}, 0\right)$

Relative Max $\left(-\frac{1}{8}, f\left(-\frac{1}{8}\right)\right) = \left(-\frac{1}{8}, \frac{43}{20}\right)$

Relative Min $(0, f(0)) = (0, 2)$

2. $f(x) = x^{\frac{8}{3}} - 4x^{\frac{2}{3}} + 4$

i. Compute $f'(x)$ and find critical numbers

$$f'(x) = \frac{8}{3}x^{\frac{5}{3}} - \frac{8}{3}x^{-\frac{1}{3}} = \frac{8x^{\frac{5}{3}}}{3} - \frac{8}{3x^{\frac{1}{3}}} = \frac{8x^{\frac{5}{3}}}{3}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{8}{3x^{\frac{1}{3}}} = \frac{8x^{\frac{6}{3}}}{3x^{\frac{1}{3}}} - \frac{8}{3x^{\frac{1}{3}}} = \frac{8x^2-8}{3x^{\frac{1}{3}}}$$

i.e., $f'(x) = \frac{8x^2-8}{3x^{\frac{1}{3}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{8x^2-8}{3x^{\frac{1}{3}}} = 0$

$\Rightarrow 8x^2 - 8 = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x+1)(x-1) = 0$

$\Rightarrow x = -1;\ x = 1$ critical numbers

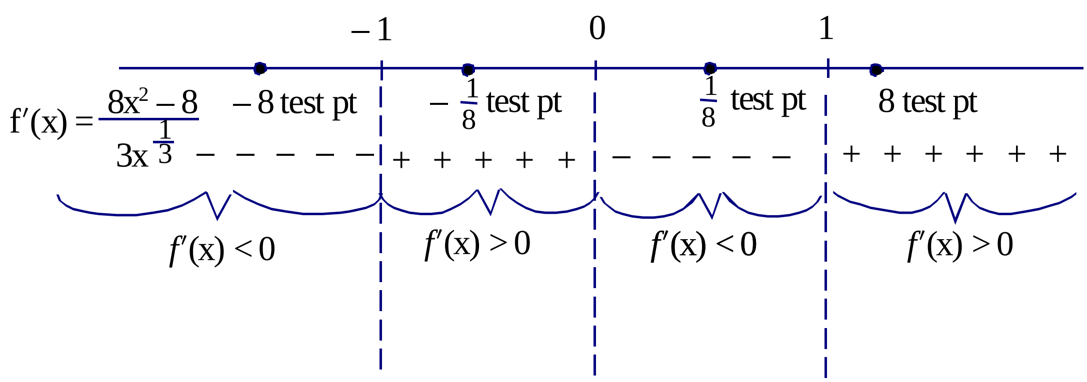
b. "Type b" $(f'(c)$ undefined$)$

Set denominator $3x^{\frac{1}{3}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



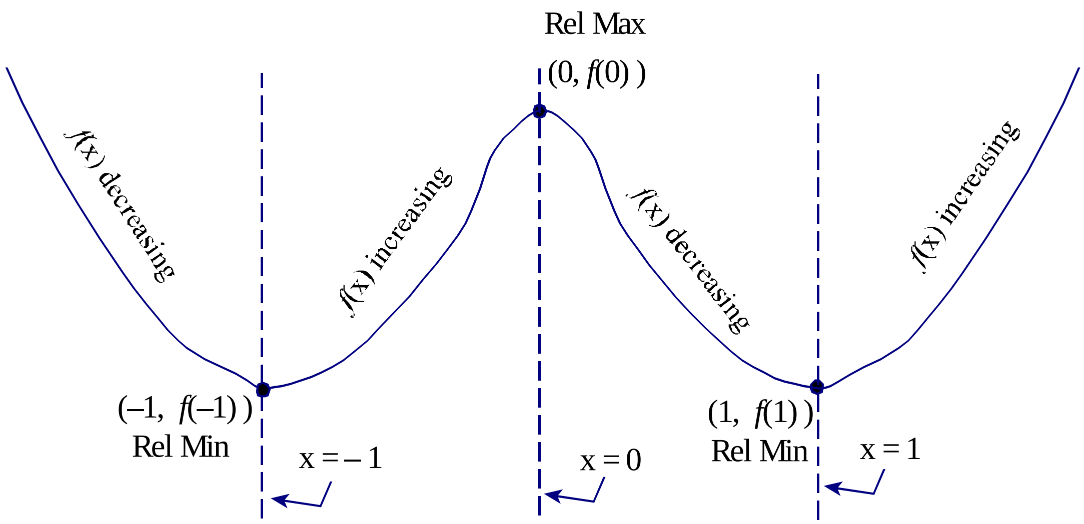
$f(x)$ is **increasing** on the intervals $(-1, 0)$ and $(1, \infty)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals $(-\infty, -1)$ and $(0, 1)$

(Because $f'(x)$ is negative on this interval)

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals and $(-1, 0)$ and $(1, \infty)$

$f(x)$ is **decreasing** on the interval $(-\infty, -1)$ and $(0, 1)$

Relative Max $(0, f(0)) = (0, 4)$

Relative Mins $(-1, f(-1)) = (-1, 1)$ and $(1, f(1)) = (1, 1)$

3. $f(x) = x^{\frac{9}{5}} - 9x^{\frac{4}{5}} + 1$

i. Compute $f'(x)$ and find critical numbers

$f'(x) = \frac{9}{5}x^{\frac{4}{5}} - \frac{36}{5}x^{-\frac{1}{5}} = \frac{9x^{\frac{4}{5}}}{5} - \frac{36}{5x^{\frac{1}{5}}} = \frac{9x^{\frac{4}{5}}}{5}\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}} - \frac{36}{5x^{\frac{1}{5}}} = \frac{9x}{5x^{\frac{1}{5}}} - \frac{36}{5x^{\frac{1}{5}}} = \frac{9x-36}{5x^{\frac{1}{5}}}$

i.e., $f'(x) = \frac{9x-36}{5x^{\frac{1}{5}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{9x-36}{5x^{\frac{1}{5}}} = 0$

$\Rightarrow 9x - 36 = 0$

$\Rightarrow x - 4 = 0$

$\Rightarrow x = 4$ critical number

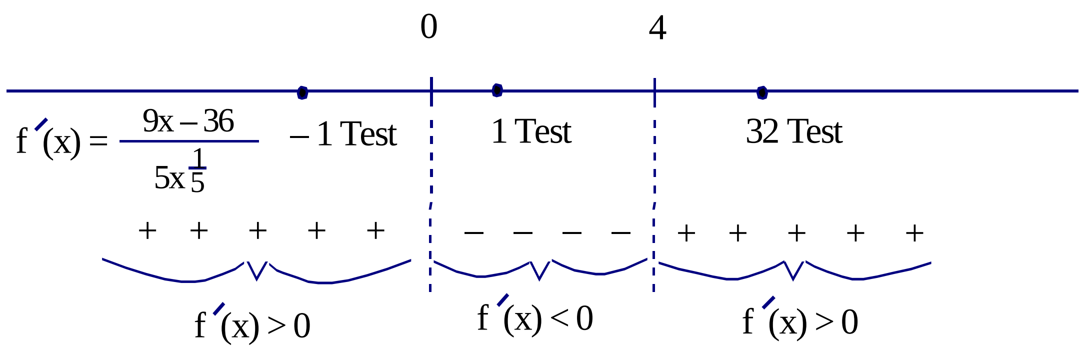
b. "Type b" $(f'(c)$ undefined$)$

Set denominator $5x^{\frac{1}{5}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



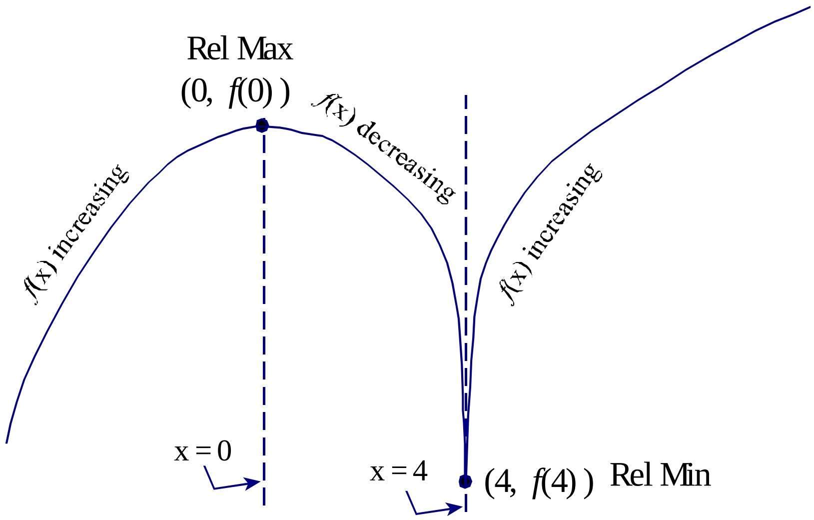
$f(x)$ is **increasing** on the intervals $(-\infty, -1)$ and $(0, \infty)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals $(-1, 0)$

(Because $f'(x)$ is negative on this interval.

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals $(-\infty, -4)$ and $(0, \infty)$

$f(x)$ is **decreasing** on the interval $(-4, 0)$

Relative Max $\quad (-4, f(-4)) = \left(-4, (-4)^{\frac{9}{5}} - 9(-4)^{\frac{4}{5}} + 1\right)$

Relative Min $\quad (0, f(0)) = (0, 1)$

4. $f(x) = 2x^{\frac{14}{5}} - 7x^{\frac{4}{5}} - 2$

1. Compute $f'(x)$ and find the critical numbers

$$f'(x) = \frac{28}{5}x^{\frac{9}{5}} - \frac{28}{5}x^{-\frac{1}{5}} = \frac{28x^{\frac{9}{5}}}{5} - \frac{28}{5x^{\frac{1}{5}}} = \frac{28x^{\frac{9}{5}}}{5}\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}} - \frac{28}{5x^{\frac{1}{5}}} = \frac{28x^2-28}{5x^{\frac{1}{5}}}$$

i.e., $f'(x) = \frac{28x^2-28}{5x^{\frac{1}{5}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = 0$ and solve for x

$\Rightarrow f'(x) = \frac{28x^2-28}{5x^{\frac{1}{5}}} = 0$

$\Rightarrow 28x^2 - 28 = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x+1)(x-1) = 0$

$\Rightarrow x = -1$ and $x = 1$ are critical numbers.

b. "Type b" $(f'(c)$ is undefined$)$

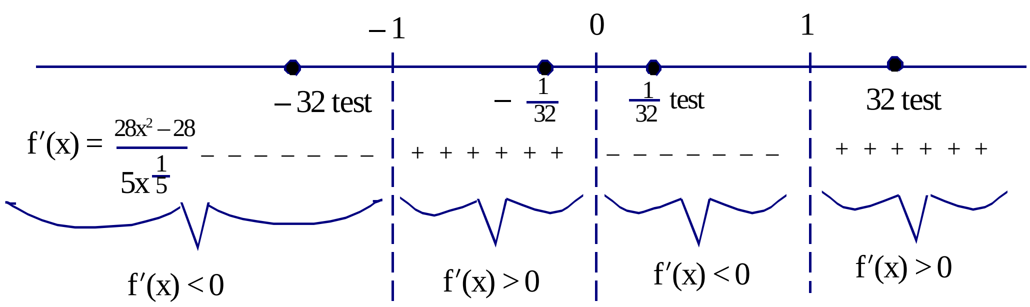
Look for x-value that causes division by zero.

$\Rightarrow 5x^{\frac{1}{5}} = 0$

$\Rightarrow x = 0$ "type b" critical number

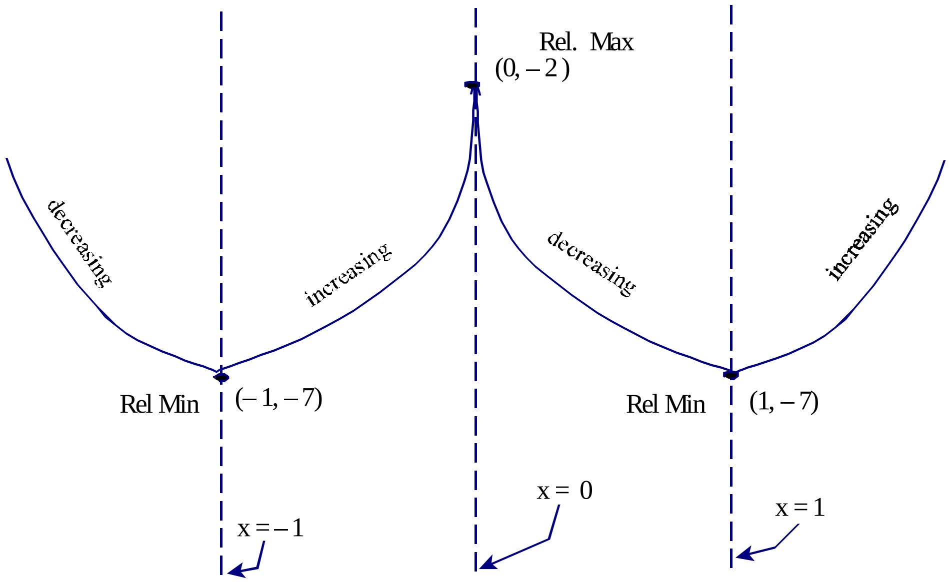
2. Draw a "sign graph" of $f'(x)$, using the critical numbers to partition the x-axis

3. Pick a "test point" from each interval to plug into $f'(x)$



> $f(x)$ is **increasing** on the interval(s) $(-1, 0)$ and $(1, \infty)$
>
> (because $f'(x)$ is positive on these intervals)
>
> $f(x)$ is **decreasing** on the interval(s) $(-\infty, -1)$ and $(0, 1)$
>
> (because $f'(x)$ is negative on that interval)

4. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



Rel Minimums: $(-1, f(-1)) = (-1, -7)$

and $(1, f(1)) = (1, -7)$

Rel Maximum: $(0, f(0)) = (0, -2)$

5. $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$

i. Compute $f'(x)$ and find critical numbers

$$f'(x) = \frac{10}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}} = \frac{10x^{\frac{2}{3}}}{3} - \frac{10}{3x^{\frac{1}{3}}} = \frac{10x^{\frac{2}{3}}}{3}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{10}{3x^{\frac{1}{3}}} = \frac{10x^{\frac{3}{3}}}{3} - \frac{10}{3x^{\frac{1}{3}}} = \frac{10x-10}{3x^{\frac{1}{3}}}$$

i.e. $f'(x) = \frac{10x-10}{3x^{\frac{1}{3}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{10x-10}{3x^{\frac{1}{3}}} = 0$

$\Rightarrow 10x - 10 = 0$

$\Rightarrow 10x = 10$

$\Rightarrow x = 1$ critical number

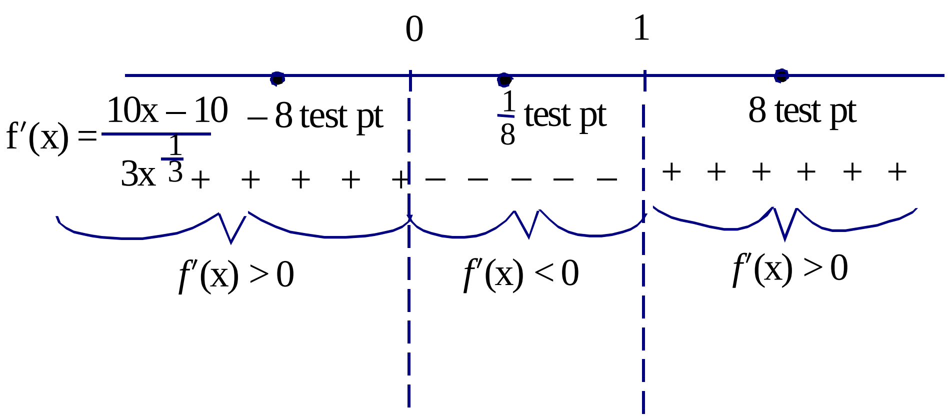
b. "Type b" ($f'(c)$ undefined)

Set denominator $3x^{\frac{1}{3}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



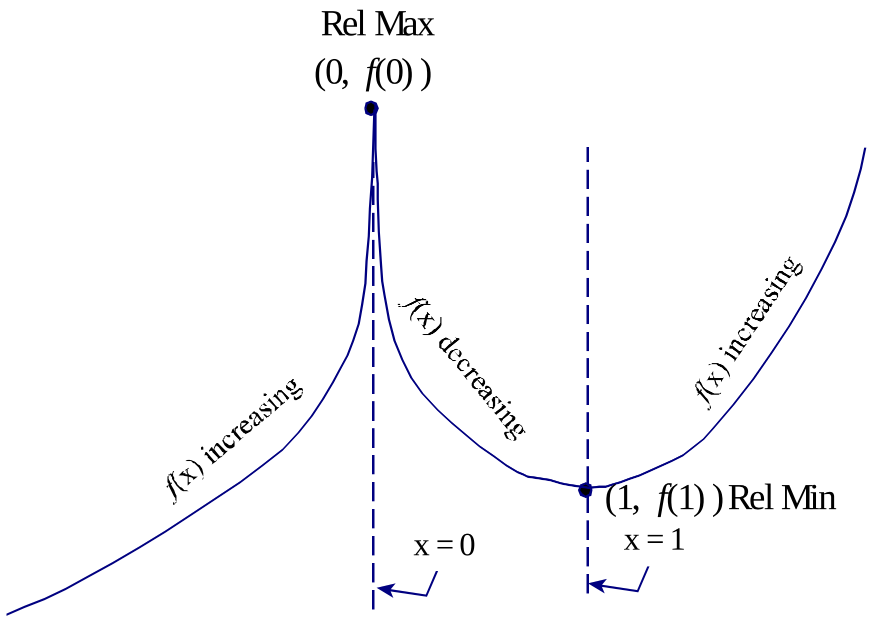
$f(x)$ is **increasing** on the intervals $(-\infty, 0)$ and $(1, \infty)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals and $(0, 1)$

(Because $f'(x)$ is negative on this interval)

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals and $(-\infty, 0)$ and $(1, \infty)$

$f(x)$ is **decreasing** on the interval $(0, 1)$

Relative Max $(0, f(0)) = (0, 0)$

Relative Min $(1, f(1)) = (1, -3)$

6. $f(x) = x^{\frac{8}{3}} - x^{\frac{2}{3}} + 5$

i. Compute $f'(x)$ and find critical numbers

$$f'(x) = \frac{8}{3}x^{\frac{5}{3}} - \frac{2}{3}x^{-\frac{1}{3}} = \frac{8x^{\frac{5}{3}}}{3} - \frac{2}{3x^{\frac{1}{3}}} = \frac{8x^{\frac{5}{3}}}{3}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{2}{3x^{\frac{1}{3}}} = \frac{8x^{\frac{6}{3}}}{3x^{\frac{1}{3}}} - \frac{2}{3x^{\frac{1}{3}}} = \frac{8x^2-2}{3x^{\frac{1}{3}}}$$

i.e., $f'(x) = \frac{8x^2-2}{3x^{\frac{1}{3}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{8x^2-2}{3x^{\frac{1}{3}}} = 0$

$\Rightarrow 8x^2 - 2 = 0$

$\Rightarrow 4x^2 - 1 = 0$

$\Rightarrow (2x+1)(2x-1) = 0$

$\Rightarrow 2x + 1 = 0 \quad$ or $\quad 2x - 1 = 0$

$\Rightarrow x = -\frac{1}{2};\ x = \frac{1}{2}$ critical numbers

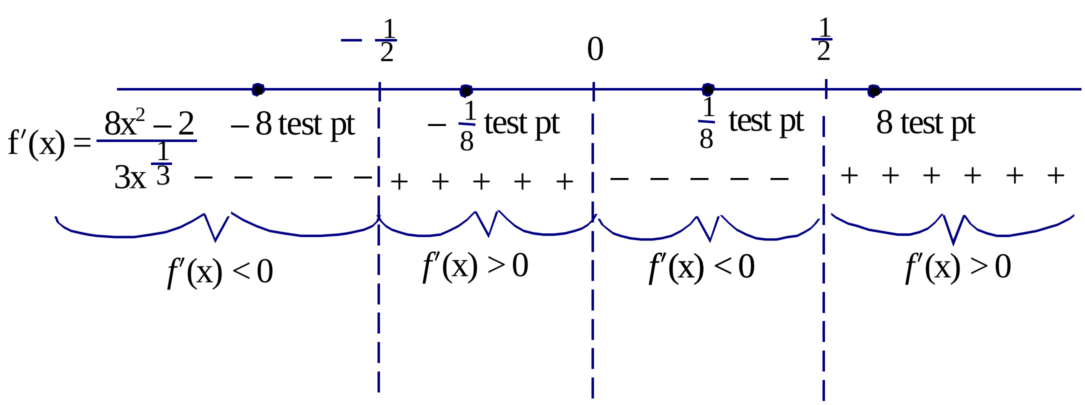
b. "Type b" $(f'(c)$ undefined$)$

Set denominator $3x^{\frac{1}{3}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



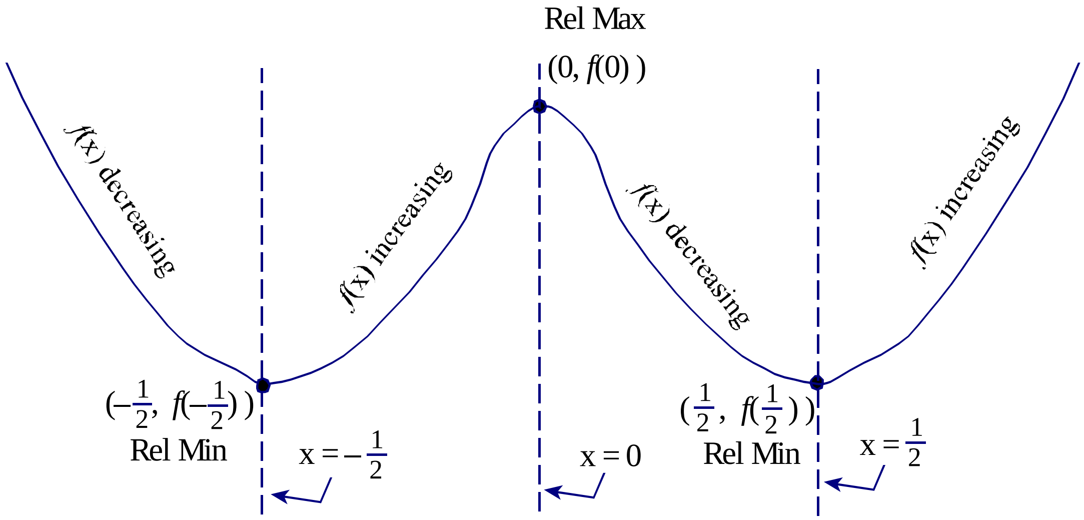
$f(x)$ is **increasing** on the intervals $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, \infty\right)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals $\left(-\infty, -\frac{1}{2}\right)$ and $\left(0, \frac{1}{2}\right)$

(Because $f'(x)$ is negative on this interval)

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals and $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, \infty\right)$

$f(x)$ is **decreasing** on the interval $\left(-\infty, -\frac{1}{2}\right)$ and $\left(0, \frac{1}{2}\right)$

Relative Max $\quad (0, f(0)) = (0, 5)$

Relative Mins $\quad \left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right) = \left(-\frac{1}{2}, \left(-\frac{1}{2}\right)^{\frac{8}{3}} - \left(-\frac{1}{2}\right)^{\frac{2}{3}} + 5\right)$ and $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \left(\frac{1}{2}\right)^{\frac{8}{3}} - \left(\frac{1}{2}\right)^{\frac{2}{3}} + 5\right)$

7. $f(x) = \frac{1}{7}x^{\frac{14}{5}} - 2x^{\frac{4}{5}} + 1$

1. Compute $f'(x)$ and find the critical numbers

$f'(x) = \frac{2}{5}x^{\frac{9}{5}} - \frac{8}{5}x^{-\frac{1}{5}} = \frac{2x^{\frac{9}{5}}}{5} - \frac{8}{5x^{\frac{1}{5}}} = \frac{2x^{\frac{9}{5}}}{5}\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}} - \frac{8}{5x^{\frac{1}{5}}} = \frac{2x^2-8}{5x^{\frac{1}{5}}}$

i.e., $f'(x) = \frac{2x^2-8}{5x^{\frac{1}{5}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = 0$ and solve for x

$\Rightarrow f'(x) = \frac{2x^2-8}{5x^{\frac{1}{5}}} = 0$

$\Rightarrow 2x^2 - 8 = 0$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow (x+2)(x-2) = 0$

$\Rightarrow x = -2$ and $x = 2$ are critical numbers.

b. "Type b" $(f'(c)$ is undefined)

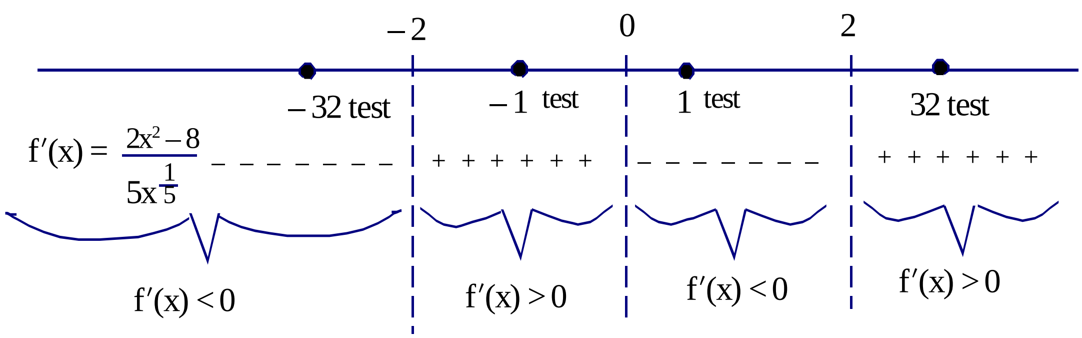
Look for x-value that causes division by zero.

$\Rightarrow 5x^{\frac{1}{5}} = 0$

$\Rightarrow x = 0$ "type b" critical number

2. Draw a "sign graph" of $f'(x)$, using the critical numbers to partition the x-axis

3. Pick a "test point" from each interval to plug into $f'(x)$



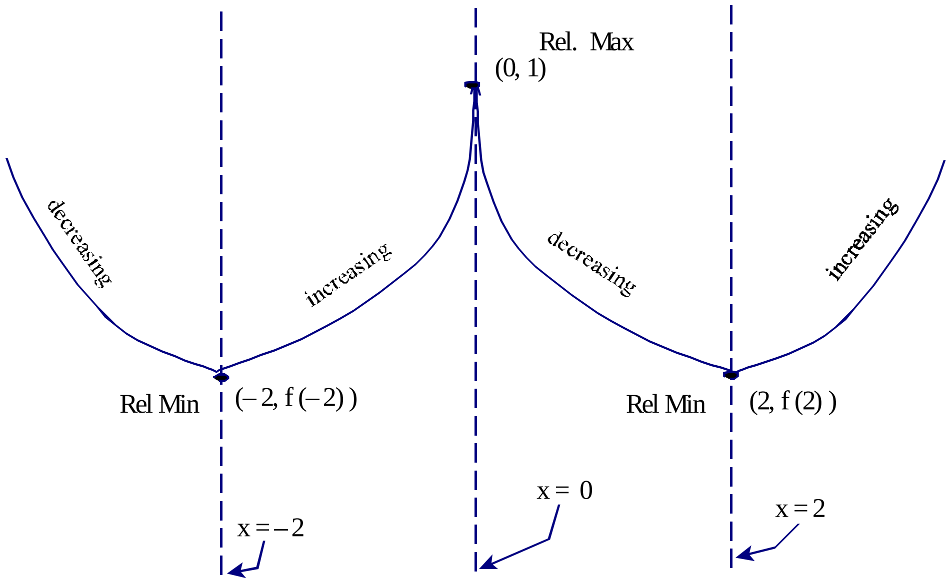
$f(x)$ is **increasing** on the interval(s) $(-\infty, 0)$ and $(4, \infty)$

(because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the interval(s) $(0, 4)$

(because $f'(x)$ is negative on that interval)

4. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



Rel Minimums: $(-2, f(-2)) = \left(-2, \frac{1}{7}(-2)^{\frac{14}{5}} - 2(-2)^{\frac{4}{5}} + 1\right)$

and $(2, f(2)) = \left(2, \frac{1}{7}(2)^{\frac{14}{5}} - 2(2)^{\frac{4}{5}} + 1\right)$

Rel Maximum: $(0, f(0)) = (0, 1)$

8. $f(x) = 4x^{\frac{9}{5}} + 9x^{\frac{4}{5}} + \frac{1}{2}$

i. Compute $f'(x)$ and find critical numbers

$$f'(x) = \frac{36}{5}x^{\frac{4}{5}} + \frac{36}{5}x^{-\frac{1}{5}} = \frac{36x^{\frac{4}{5}}}{5} + \frac{36}{5x^{\frac{1}{5}}} = \frac{36x^{\frac{4}{5}}}{5}\frac{x^{\frac{1}{5}}}{x^{\frac{1}{5}}} + \frac{36}{5x^{\frac{1}{5}}} = \frac{36x}{5x^{\frac{1}{5}}} + \frac{36}{5x^{\frac{1}{5}}} = \frac{36x+36}{5x^{\frac{1}{5}}}$$

i.e., $f'(x) = \frac{36x+36}{5x^{\frac{1}{5}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{36x+36}{5x^{\frac{1}{5}}} = 0$

$\Rightarrow 36x + 36 = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$ critical numbers

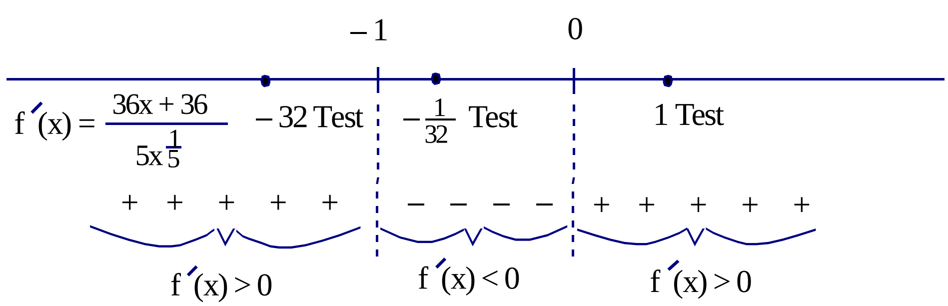
b. "Type b" $(f'(c)$ undefined)

Set denominator $5x^{\frac{1}{5}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



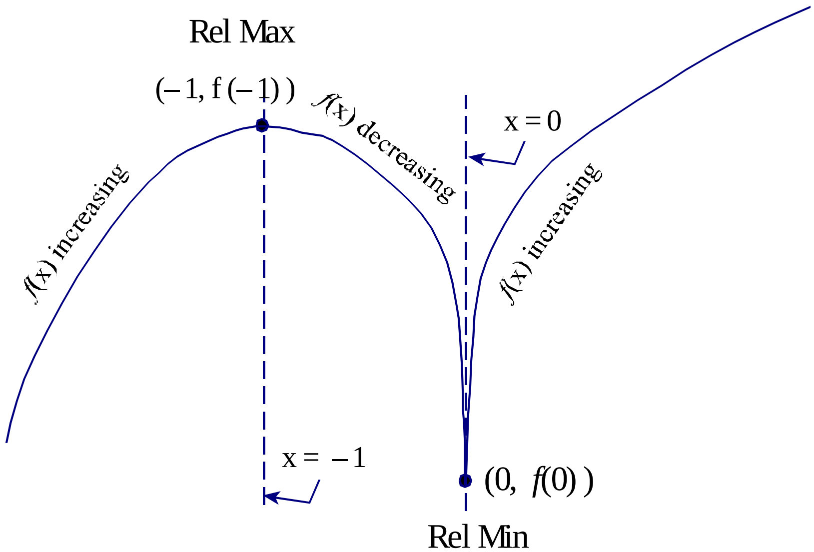
$f(x)$ is **increasing** on the intervals $(-\infty, -1)$ and $(0, \infty)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals $(-1, 0)$

(Because $f'(x)$ is negative on this interval.

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals $(-\infty, -1)$ and $(0, \infty)$

$f(x)$ is **decreasing** on the interval $(-1, 0)$

Relative Max $(-1, f(-1)) = \left(-1, \frac{11}{5}\right)$

Relative Min $(0, f(0)) = \left(0, \frac{1}{2}\right)$

9. $f(x) = \frac{1}{2}x^{\frac{8}{3}} - 8x^{\frac{2}{3}} - 2$

i. Compute $f'(x)$ and find critical numbers

$f'(x) = \frac{4}{3}x^{\frac{5}{3}} - \frac{16}{3}x^{-\frac{1}{3}} = \frac{4x^{\frac{5}{3}}}{3} - \frac{16}{3x^{\frac{1}{3}}} = \frac{4x^{\frac{5}{3}}}{3}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{16}{3x^{\frac{1}{3}}} = \frac{4x^{\frac{6}{3}}}{3x^{\frac{1}{3}}} - \frac{16}{3x^{\frac{1}{3}}} = \frac{4x^2-16}{3x^{\frac{1}{3}}}$

i.e., $f'(x) = \frac{4x^2-16}{3x^{\frac{1}{3}}}$

a. "Type a" $(f'(c) = 0)$

Set $f'(x) = \frac{4x^2-16}{3x^{\frac{1}{3}}} = 0$

$\Rightarrow 4x^2 - 16 = 0$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow (x+2)(x-2) = 0$

$\Rightarrow x = -2$; $x = 2$ critical numbers

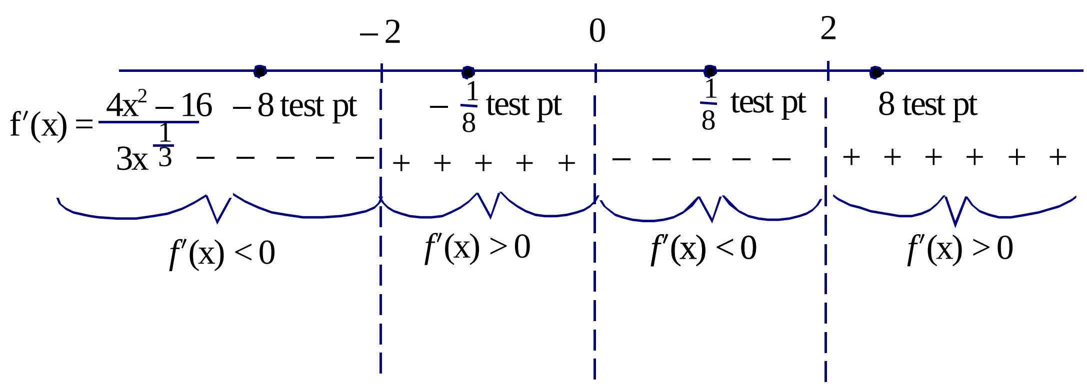
b. "Type b" $(f'(c)$ undefined$)$

Set denominator $3x^{\frac{1}{3}} = 0$

$\Rightarrow x = 0$ critical number

ii. Draw a sign graph of $f'(x)$, using the critical numbers to partition the x-axis

iii. From each interval select a "test point" to plug into $f'(x)$



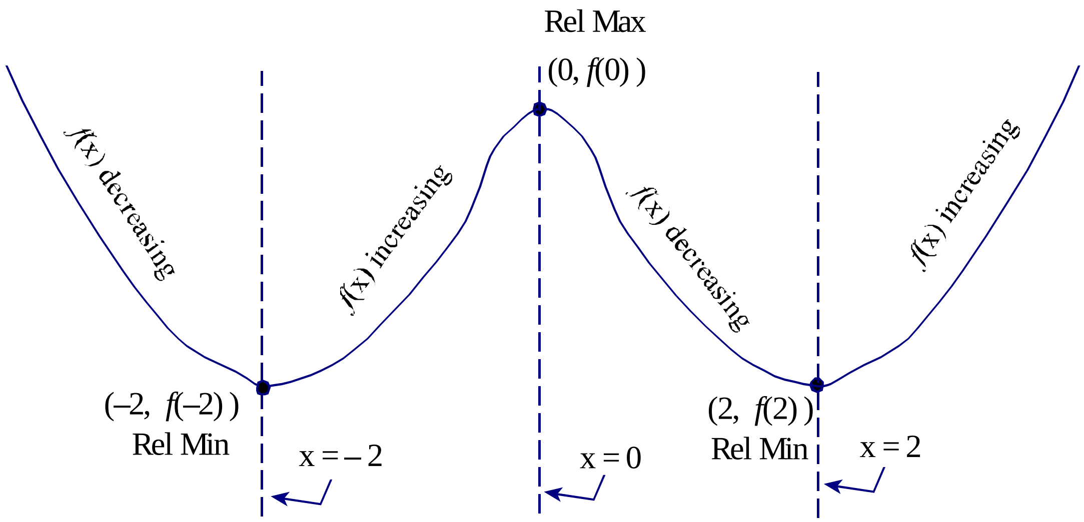
$f(x)$ is **increasing** on the intervals $(-2, 0)$ and $(2, \infty)$

(Because $f'(x)$ is positive on these intervals)

$f(x)$ is **decreasing** on the intervals $(-\infty, -2)$ and $(0, 2)$

(Because $f'(x)$ is negative on this interval)

iv. To find the relative maxes and mins, sketch a rough graph of $f(x)$.



$f(x)$ is **increasing** on the intervals and $(-2, 0)$ and $(2, \infty)$

$f(x)$ is **decreasing** on the interval $(-\infty, -2)$ and $(0, 2)$

Relative Max $\quad (0, f(0)) = (0, -2)$

Relative Mins $\quad (-2, f(-2)) = \left(-2, \ \frac{1}{2}(-2)^{\frac{8}{3}} - 8(-2)^{\frac{2}{3}} - 2\right)$

and

$(2, f(2)) = \left(2, \ \frac{1}{2}(2)^{\frac{8}{3}} - 8(2)^{\frac{2}{3}} - 2\right)$