MTH 1125 Test #1 - (12 pm class) - Solutions

FALL 2022

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2+4x-8}{x^2+2x+5} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3} \frac{x^2+4x-8}{x^2+2x+5} = \frac{(3)^2+4(3)-8}{(3)^2+2(3)+5} = \frac{13}{20}$$

i.e., $\lim_{x\to 3} \frac{x^2+4x-8}{x^2+2x+5} = \frac{13}{20}$

2. Compute: $\lim_{x\to 3} \frac{x^2-8x+15}{2x^2-5x-3} =$

$$\lim_{x\to 3} \frac{x^2-8x+15}{2x^2-5x-3} = \frac{(3)^2-8(3)+15}{2(3)^2-5(3)-3} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3} \frac{x^2-8x+15}{2x^2-5x-3} = \lim_{x\to 3} \frac{(x-3)(x-5)}{(2x+1)(x-3)} = \lim_{x\to 3} \frac{x-5}{2x+1} = \frac{(3)-5}{2(3)+1} = \frac{-2}{7} = -\frac{2}{7}$$

i.e., $\lim_{x\to 3} \frac{x^2-8x+15}{2x^2-5x-3} = -\frac{2}{7}$

3. Compute: $\lim_{x\to\,-4} \frac{x^2+2x-9}{x^2+2x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to\,-4} \frac{x^2+2x-9}{x^2+2x-8} = \frac{(-4)^2+2(-4)-9}{(-4)^2+2(-4)-8} = \frac{-1}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\,-4^-} \frac{x^2+2x-9}{x^2+2x-8} = \lim_{x\to\,-4^-} \frac{x^2+2x-9}{(x+4)(x-2)} = \frac{-1}{(-\varepsilon)(-6)} = \frac{1}{(-\varepsilon)(6)} = \frac{\left(\frac{1}{6}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{aligned} & x \to -4^- \\ \Rightarrow\ & x < -4 \\ \Rightarrow\ & x + 4 < 0 \end{aligned}$$

$$\lim_{x\to\,-4^+} \frac{x^2+2x-9}{x^2+2x-8} = \lim_{x\to\,-4^+} \frac{x^2+2x-9}{(x+4)(x-2)} = \frac{-1}{(+\varepsilon)(-6)} = \frac{1}{(+\varepsilon)(6)} = \frac{\left(\frac{1}{6}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{aligned} & x \to -4^+ \\ \Rightarrow\ & x > -4 \\ \Rightarrow\ & x + 4 > 0 \end{aligned}$$

Since the one-sided limits are not equal, $\lim_{x\to\,-4} \frac{x^2+2x-9}{x^2+2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\,-\infty} \frac{9x^4+7x-5}{4x^5+6x^3-8x} =$

$$\lim_{x\to\,-\infty} \frac{9x^4+7x-5}{4x^5+6x^3-8x} = \lim_{x\to\,-\infty} \frac{9x^4}{4x^5} = \lim_{x\to\,-\infty} \frac{9}{4x} = 0$$

i.e., $\lim_{x\to\,-\infty} \frac{9x^4+7x-5}{4x^5+6x^3-8x} = 0$

5. $f(x) = \frac{x^2-2x-3}{x^2-8x+16} = \frac{x^2-2x-3}{(x-4)^2}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow (x-4)^2 = 0$

$\Rightarrow (x-4) = 0$

$\Rightarrow x = 4$ is a possible vertical asymptote.

2. Compute the one-sided limits.

$\lim_{x\to\ 4^-} \frac{x^2-2x-3}{(x-4)^2} = \lim_{x\to\ -4^-} \frac{5}{(-\varepsilon)^2} = \frac{5}{(\varepsilon)^2} = +\infty$

$$\boxed{\begin{array}{ll} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \end{array}}$$

$\lim_{x\to\ 4^+} \frac{x^2-2x-3}{(x-4)^2} = \lim_{x\to\ -4^+} \frac{5}{(+\varepsilon)^2} = \frac{5}{(\varepsilon)^2} = +\infty$

$$\boxed{\begin{array}{ll} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \end{array}}$$

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-2x-3}{(x-4)^2} = \lim_{x\to-\infty} \frac{x^2-2x-3}{x^2-8x+16} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

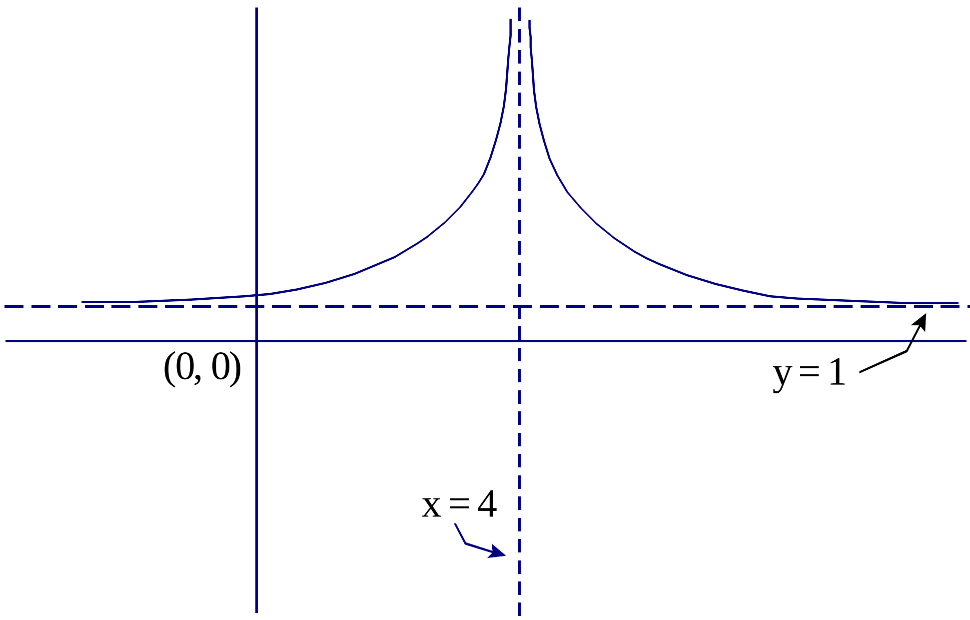
$\lim_{x\to+\infty} \frac{x^2-2x-3}{(x-4)^2} = \lim_{x\to+\infty} \frac{x^2-2x-3}{x^2-8x+16} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to 4^-} \frac{x^2-2x-3}{(x-4)^2} = +\infty$	$\lim_{x\to -\infty} \frac{x^2-2x-3}{(x-4)^2} = 1$
$\lim_{x\to 4^+} \frac{x^2-2x-3}{(x-4)^2} = +\infty$	$\lim_{x\to +\infty} \frac{x^2-2x-3}{(x-4)^2} = 1$

Graph $f(x) = \frac{x^2-2x-3}{x^2-8x+16} = \frac{x^2-2x-3}{(x-4)^2}$



6. Compute: $\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} = \lim_{x\to 8} \frac{\sqrt{(8)+1}-3}{(8)-8} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} = \lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} \cdot \frac{\sqrt{x+1}+3}{\sqrt{x+1}+3} = \lim_{x\to 8} \frac{\left(\sqrt{x+1}\right)^2-(3)^2}{(x-8)\left[\sqrt{x+1}+3\right]}$$

$$= \lim_{x\to 8} \frac{(x+1)-9}{(x-8)\left[\sqrt{x+1}+3\right]} = \lim_{x\to 8} \frac{(x-8)}{(x-8)\left[\sqrt{x+1}+3\right]} = \lim_{x\to 8} \frac{1}{\left[\sqrt{x+1}+3\right]}$$

$$= \frac{1}{\left[\sqrt{(8)+1}+3\right]} = \frac{1}{3+3} = \frac{1}{6}$$

i.e., $\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} = \frac{1}{6}$

7.

$x =$	$f(x) =$
-10	1.5
-100	1.9
$-1,000$	1.99
$-10,000$	1.999
$-100,000$	1.9999

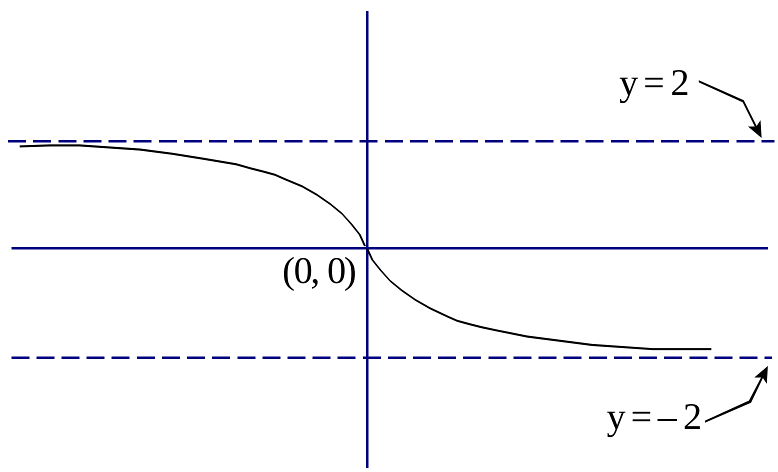
$x =$	$f(x) =$
10	-1.5
100	-1.9
$1,000$	-1.99
$10,000$	-1.99
$100,000$	-1.9999

Based on the information in the table above, compute/do the following:

(a) $\lim_{x \to -\infty} f(x) = 2$

(b) $\lim_{x \to +\infty} f(x) = -2$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 4$. (Justify Your Answer)

$$f(x) = \begin{cases} 3x - 3 & \text{for } x < 4 \\ 9 & \text{for } x = 4 \\ x^2 - 7 & \text{for } x > 4 \end{cases}$$

First of all, let's recognize that $f(x)$ will be continuous at the point $x = 4$ exactly when $\lim_{x \to 4} f(x) = f(4)$.

So we should compute: $\lim_{x \to 4} f(x)$

The problem is that $f(x)$ is defined differently for $x < 4$ than it is for $x > 4$.

So we must compute the one sided limits as $x \to 4$

Observe: As $x \to 4^-$, $x < 4$.

Therefore: $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (3x - 3) = 3(4) - 3 = 9$

Also: As $x \to 4^+$, $x > 4$.

Therefore: $\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (x^2 - 7) = (4)^2 - 7 = 9$

Since the one-sided limits are equal, $\lim_{x \to 4} f(x)$ exists, and is equal to the common value of the one-sided limits.

i.e., $\lim_{x \to 4} f(x) = 9$

Finally, note that $f(4) = 9$

$\Rightarrow \lim_{x \to 4} f(x) = f(4)$

Hence, $f(x)$ is continuous at the point $x = 4$