MTH 3318 Test #1

Fall 2024

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Instructions. Fully document your work.

For problems 1 - 2 prove one using Mathematical Induction.

1. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

   i.e. $\sum_{i=1}^{n} (2i - 1) = n^2$

2. $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

   i.e. $\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$

For problems 3 - 5 prove one using Mathematical Induction.

3. $1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$

   i.e., $\sum_{i=1}^{n} (4i - 3) = 2n^2 - n$

4. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

   i.e. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

5. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \ldots + n^3$ all natural numbers, n.

For problems 6 - 7, prove one using Mathematical Induction:

6. $n\left(n+1\right)$ is divisible by 2 for all natural numbers, n.

7. Given that $\frac{d}{dx}\left[x^0\right]=0$ and $\frac{d}{dx}\left[x^1\right]=1$, prove that $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$. You may use the product rule: $\frac{d}{dx}\left[f\left(x\right)g\left(x\right)\right]=f'\left(x\right)g\left(x\right)+g'\left(x\right)f\left(x\right).$

For problems 8 - 9, prove one using Mathematical Induction:

8. $(1+x)^n \geq 1+nx$ for any natural number n and any real number $x \geq -1$.

9. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that: $|x_1 + x_2 + x_3 + \ldots + x_n| \leq |x_1|+|x_2|+|x_3|+\ldots+|x_n|$ (the General Triangle Inequality).