MTH 1126 - Test #1 - Solutions

Spring 2005

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\int \sec(x^3)\tan(x^3)\,x^2\,dx =$

   1. Is u-sub appropriate?

      a. Is there a composite function?

      Yes. $\sec(x^3)\tan(x^3)$

      Let $u = x^3$

      b. Is there an "approximate function/derivative pair"?

      Yes. $x^3 \to x^2$

      Let $u = x^3$

   2. Compute du

$$\begin{array}{rrcl} & u & = & x^3 \\ \Rightarrow & \frac{du}{dx} & = & 3x^2 \\ \Rightarrow & du & = & 3x^2dx \\ \Rightarrow & \frac{1}{3}du & = & x^2dx \end{array}$$

   3. Analyze in terms of u and du.

$$\int \underbrace{\sec\left(x^3\right)\tan\left(x^3\right)}_{\sec(u)\tan(u)} \underbrace{x^2\,dx}_{\frac{1}{3}du} = \int \sec(u)\tan(u)\,\tfrac{1}{3}du = \tfrac{1}{3}\int \sec(u)\tan(u)\,du$$

   4. Integrate in terms of u

$$\tfrac{1}{3}\int \sec(u)\tan(u)\,du = \tfrac{1}{3}\sec(u) + C$$

   5. Re-write in terms of x

$$\int \sec(x^3)\tan(x^3)\,x^2\,dx = \tfrac{1}{3}\sec(x^3) + C$$

2. Use the "$f-g$" method to compute the area bounded by the graphs of $f(x) = 2x^2 - 4$ and $g(x) = x^2$.

First, graph the functions and find the points of intersection.

To find the points of intersection, set $f(x) = g(x)$.

$f(x) = 2x^2 - 4 = x^2 = g(x)$

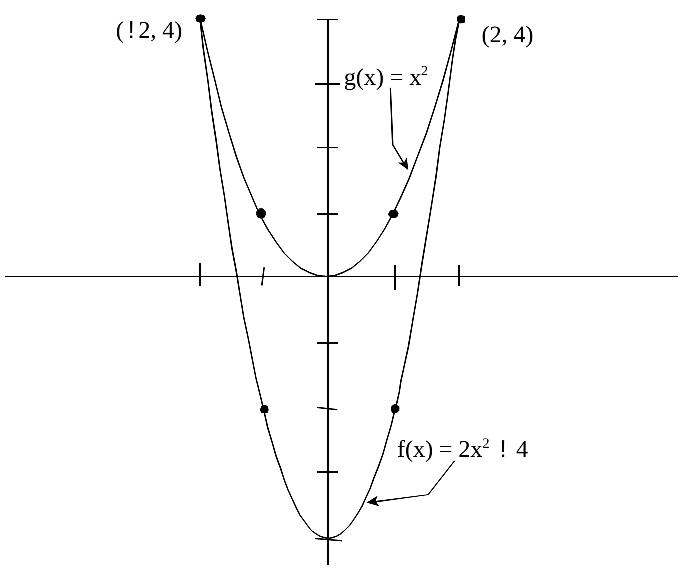
$\Rightarrow 2x^2 - 4 = x^2$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow (x+2)(x-2) = 0$

$\Rightarrow x = -2;\ x = 2$

The points of intersection are: $(-2, 4)$ and $(2, 4)$.



Since $x^2 \geq 2x^2 - 4$ over the interval $[-2, 2]$, the area of the bounded region is given by

$\int_{-2}^{2} [(x^2) - (2x^2 - 4)]\, dx = \int_{-2}^{2} (-x^2 + 4)\, dx = \left[-\frac{1}{3}x^3 + 4x\right]_{-2}^{2} =$

$\left(-\frac{1}{3}(2)^3 + 4(2)\right) - \left(-\frac{1}{3}(-2)^3 + 4(-2)\right) = \frac{32}{3}$

3. Compute: $\int \left(2x^5 - 8\right)^{15} 3x^4 \; dx =$

   1. Is u-sub appropriate?

      a. Is there a composite function?

      Yes. $\left(2x^5 - 8\right)^{15}$

      Let $u = 2x^5 - 8$

      b. Is there an "approximate function/derivative pair"?

      Yes. $(2x^5 - 8) \rightarrow 3x^4$

      Let $u = 2x^5 - 8$

   2. Compute du

$$\boxed{\begin{array}{rrcl} & u & = & 2x^5 - 8 \\ \Rightarrow & \frac{du}{dx} & = & 10x^4 \\ \Rightarrow & du & = & 10x^4 dx \\ \Rightarrow & \frac{1}{10} du & = & x^4 dx \\ \Rightarrow & \frac{3}{10} du & = & 3x^4 dx \end{array}}$$

   3. Analyze in terms of u and du.

$$\int \underbrace{\left(2x^5 - 8\right)^{15}}_{u^{15}} \underbrace{3x^4 \; dx}_{\frac{3}{10} du} = \int u^{15} \tfrac{3}{10} du = \tfrac{3}{10} \int u^{15} du$$

   4. Integrate in terms of u

$$\tfrac{3}{10} \int u^{15} du = \tfrac{3}{10} \tfrac{u^{16}}{16} + C = \tfrac{3}{160} u^{16} + C$$

   5. Re-write in terms of x

$$\int \left(2x^5 - 8\right)^{15} 3x^4 \; dx = \tfrac{3}{160} \left(2x^5 - 8\right)^{16} + C$$

4. Find the area bounded by the graphs of $f(x) = x^2 - 2$ and $g(x) = 2x + 1$. (Partition the proper interval, build the Riemann Sum, derive the appropriate integral.)

   1. First, graph the functions and find the points of intersection.

      To find the points of intersection, set $f(x) = g(x)$.

      $f(x) = x^2 - 2 = 2x + 1 = g(x)$

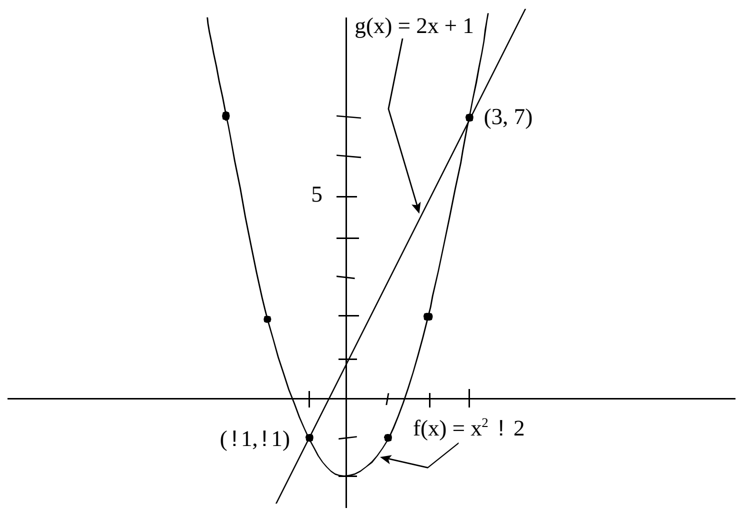
      $\Rightarrow x^2 - 2 = 2x + 1$

      $\Rightarrow x^2 - 2x - 3 = 0$

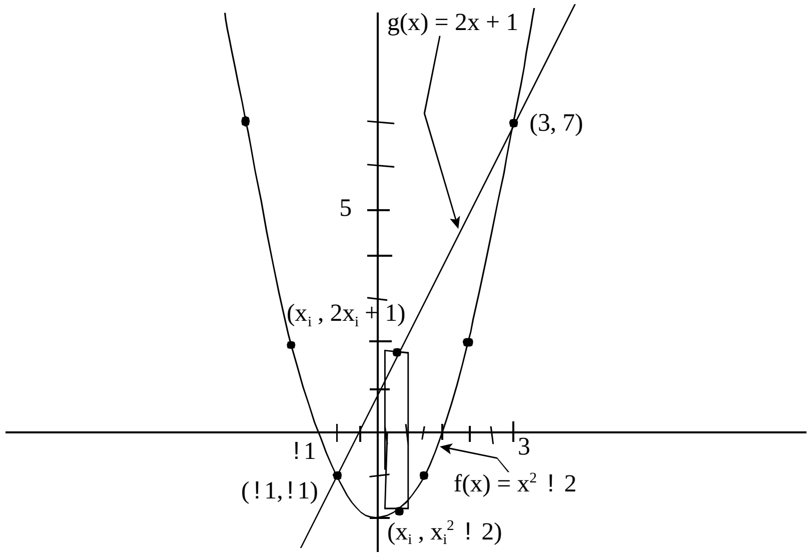
      $\Rightarrow (x+1)(x-3) = 0$

      $\Rightarrow x = -1;\ x = 3$

      Points of intersection are $(-1, -1)$ and $(3, 7)$.



   2. Partition the interval spanned by the region into sub-intervals of length Δx.
   3. Above the i^{th} subinterval, inscribe a rectangle of width Δx.



Height of the i^{th} rectangle $= [(2x_i + 1) - (x_i^2 - 2)] = 2x_i + 3 - x_i^2$

Width of the i^{th} rectangle $= \Delta x$

Area of the i^{th} rectangle $= (2x_i + 3 - x_i^2)\,\Delta x$

4. Approximate the area of the region by adding up the areas of the rectangles.

   Area $\approx \sum_{i=1}^{n} (2x_i + 3 - x_i^2)\,\Delta x$

5. Let $\Delta x \to 0$.

   Area $= \lim_{\Delta x \to 0} \sum_{i=1}^{n} (2x_i + 3 - x_i^2)\,\Delta x = \int_{-1}^{3} (2x + 3 - x^2)\,dx = \left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^{3} =$

$\left((3)^2 + 3\,(3) - \frac{(3)^3}{3}\right) - \left((-1)^2 + 3\,(-1) - \frac{(-1)^3}{3}\right) = \frac{32}{3}$

5. Compute: $\int_{-1}^{1} (2x^3+5)^3 x^2 dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes. $(2x^3+5)^3$

   Let $u = 2x^3 + 5$

   b. Is there an "approximate function/derivative pair"?

   Yes. $(2x^3+5) \rightarrow x^2$

   Let $u = 2x^3 + 5$

2. Compute du

$$\begin{array}{rrcl} & u & = & 2x^3+5 \\ \Rightarrow & \frac{du}{dx} & = & 6x^2 \\ \Rightarrow & du & = & 6x^2 dx \\ \Rightarrow & \frac{1}{6}du & = & x^2 dx \end{array}$$

When $x = -1$; $u = 2x^3 + 5 = 2(-1)^3 + 5 = 3$

When $x = 1$; $u = 2x^3 + 5 = 2(1)^3 + 5 = 7$

3. Analyze in terms of u and du. $\int_{-1}^{1} (2x^3+5)^3 x^2 dx =$

$$\int_{x=-1}^{x=1} \underbrace{\left(2x^3+5\right)^3}_{u^3} \underbrace{x^2\,dx}_{\frac{1}{6}du} = \int_{u=3}^{u=7} u^3 \tfrac{1}{6} du = \tfrac{1}{6} \int_{u=3}^{u=7} u^3 du$$

4. Integrate in terms of u

$$\tfrac{1}{6} \int_{u=3}^{u=7} u^3 du = \left[\tfrac{1}{6}\tfrac{u^4}{4}\right]_{u=3}^{u=7} = \left[\tfrac{1}{24}u^4\right]_{u=3}^{u=7} = \left(\tfrac{1}{24}(7^4)\right) - \left(\tfrac{1}{24}(3^4)\right) = \tfrac{290}{3}$$