Calc 2 - Test #2 - SOLUTIONS

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1. $\int (3x^5 - 2x^3 - 2)\,dx = 3\left[\frac{x^6}{6}\right] - 2\left[\frac{x^4}{4}\right] - 2x + C = \frac{1}{2}x^6 - \frac{1}{2}x^4 - 2x + C$

2. $\int \left(3\csc^2(x) + \sec(x)\tan(x) - \frac{1}{2}\sin(x)\right)dx = 3\left(-\cot(x)\right) + \sec(x) - \frac{1}{2}\left(-\cos(x)\right) + C$
$= -3\cot(x) + \sec(x) + \frac{1}{2}\cos(x) + C$

3. $\int \frac{4x^2}{(4x^3+6)^{\frac{3}{2}}}dx = \int \underbrace{(4x^3+6)}_{u^{-\frac{3}{2}}}{}^{-\frac{3}{2}}\underbrace{4x^2dx}_{\frac{1}{3}du}$

$$\boxed{\begin{array}{l} u = 4x^3 + 6 \\ du = 12x^2dx \\ \frac{1}{3}du = 4x^2dx \end{array}}$$

$= \int u^{-\frac{3}{2}}\left(\frac{1}{3}du\right) = \frac{1}{3}\int u^{-\frac{3}{2}}du$

$= \frac{1}{3}\left[\frac{u^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)}\right] + C$

$= \frac{1}{3}(-2)\,u^{-\frac{1}{2}} + C = -\frac{2}{3}(4x^3+6)^{-\frac{1}{2}} + C$

4. $\frac{d}{dx}\left[\underbrace{\cos}_{outer}\underbrace{(\tan(4x))}_{inner}\right] = \underbrace{-\sin(\tan(4x))}_{\text{deriv of outer; evaluated at the inner}} \cdot \underbrace{\frac{d}{dx}\left(\overbrace{\tan}^{\text{outer}}\overbrace{(4x)}^{\text{inner}}\right)}_{\text{deriv of inner}}$

$= -\sin(\tan(4x)) \cdot \underbrace{\sec^2(4x)}_{\text{deriv of outer; eval. at inner}} \cdot \underbrace{4}_{\text{deriv of inner}} = -4\sin(\tan(4x))\sec^2(4x)$

5. $\frac{d}{dx}\left[\sec^3(4x^2)\right] = \frac{d}{dx}\underbrace{\left[\left[\sec\left(4x^2\right)\right]^3\right]}_{[g(x)]^n} = \underbrace{3\left[\sec\left(4x^2\right)\right]^2}_{n[g(x)]^{n-1}} \cdot \frac{d}{dx}\underbrace{\left[\sec\left(4x^2\right)\right]}_{g'(x)}$

$= 3\left[\sec(4x^2)\right]^2 \cdot \left[\sec(4x^2)\tan(4x^2)\right](8x)$

6. $\int \underbrace{\cot\left(\frac{x}{3}\right)\csc\left(\frac{x}{3}\right)}_{\cot(u)\csc(u)} \underbrace{dx}_{3du} = \int \csc(u)\cot(u)(3du)$

$$\boxed{\begin{array}{l}\text{Let } u = \frac{x}{3} \\ \Rightarrow du = \frac{1}{3}dx \\ \Rightarrow 3du = dx\end{array}}$$

$$\begin{aligned}
&= 3\int \csc(u)\cot(u)\,du \\
&= 3\left[-\csc(u)\right] + C \\
&= -3\csc\left(\tfrac{x}{3}\right) + C
\end{aligned}$$

7. Suppose $E(x)$ is a function having the property that $E'(x) = [E(x)]^2$. Compute $\frac{d}{dx}\left[\underbrace{E}_{\text{outer}}\underbrace{\left(x^4 - 3x\right)}_{\text{inner}}\right] = \underbrace{\left[E\left(x^4 - 3x\right)\right]^2}_{\substack{\text{deriv of outer;} \\ \text{evaluated at inner}}} \cdot \underbrace{\left(4x^3 - 3\right)}_{\text{deriv of inner}}$

8. $\int \frac{\sec^2(x)}{\sqrt{\tan^3(x)}}dx = \int \left(\tan^3(x)\right)^{-\frac{1}{2}} \sec^2(x)\,dx$

$$\boxed{\begin{array}{l}\text{Let} \quad u = \tan(x) \\ \Rightarrow du = \sec^2(x)\,dx\end{array}}$$

$$\begin{aligned}
&= \int \underbrace{(\tan(x))^{-\frac{3}{2}}}_{u^{-\frac{3}{2}}}\underbrace{\sec^2(x)\,dx}_{du} = \int u^{-\frac{3}{2}}du \\
&= \frac{u^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)} + C = -2u^{-\frac{1}{2}} + C \\
&= -2(\tan(x))^{-\frac{1}{2}} + C \\
&= -\frac{2}{\sqrt{\tan(x)}} + C
\end{aligned}$$

9. $\int \underbrace{\sec(\sin(x))\tan(\sin(x))}_{\sec(u)\tan(u)}\underbrace{\cos(x)\,dx}_{du} = \int \sec(u)\tan(u)\,du$

$$\boxed{\begin{array}{l}\text{Let} \quad u = \sin(x) \\ du = \cos(x)\,dx\end{array}}$$

$$\begin{aligned}
&= \sec(u) + C \\
&= \sec(\sin(x)) + C
\end{aligned}$$