MTH 1125 - Test 2 (12pm Class) - Solutions

Fall 2017

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[\frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8\sqrt{x} + 6\right] =$

$\frac{d}{dx}\left[\frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8x^{\frac{1}{2}} + 6\right]$

$= \frac{1}{5}\left[5x^4\right] + \left[4x^3\right] + 3\left[3x^2\right] + 16\left[x\right] + 25 + 4x^{-\frac{1}{2}} + 0$

$= x^4 + 4x^3 + 9x^2 + 16x + +25 + 4x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[\frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8\sqrt{x} + 6\right] = x^4 + 4x^3 + 9x^2 + 16x + 25 + 4x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[x^6 \cot(x)\right] =$

$$\frac{d}{dx}\left[\underbrace{x^6}_{1^{st}}\underbrace{\cot(x)}_{2^{nd}}\right] = \underbrace{6x^5}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{\cot(x)}_{2^{nd}} + \underbrace{\left(-\csc^2(x)\right)}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{x^6}_{1^{st}}$$

$\frac{d}{dx}\left[x^6 \cot(x)\right] = 6x^5 \cot(x) - \csc^2(x) \cdot x^6$

3. Compute: $\frac{d}{dx}\left[\frac{6x^2-12x+5}{\sin(x)}\right] =$

$$\frac{d}{dx}\left[\frac{\overbrace{6x^2-12x+5}^{\text{top}}}{\underbrace{\sin(x)}_{\text{Bottom}}}\right] = \frac{\overbrace{(12x-12)}^{\text{top prime}} \cdot \overbrace{\sin(x)}^{\text{bottom}} - \overbrace{(\cos(x))}^{\text{bottom prime}} \cdot \overbrace{\left(6x^2-12x+5\right)}^{\text{top}}}{\underbrace{(\sin(x))^2}_{\substack{\text{bottom}\\ \text{squared}}}}$$

i.e., $\frac{d}{dx}\left[\frac{6x^2-12x+5}{\sin(x)}\right] = \frac{(12x-12)\sin(x) - \cos(x)\left(6x^2-12x+5\right)}{\sin^2(x)}$

4. Compute: $\frac{d}{dx}\left[\left(4x^{15}+2x^{10}\right)^5\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(4x^{15}+2x^{10}\right)^5\right] = \underbrace{5\left(4x^{15}+2x^{10}\right)^4}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{\left(60x^{14}+20x^9\right)}_{\substack{\text{derivative} \\ \text{of inner}}}$$

> i.e., $\frac{d}{dx}\left[\left(4x^{15}+2x^{10}\right)^5\right] = 5\left(4x^{15}+2x^{10}\right)^4\left(60x^{14}+20x^9\right)$

5. Given that $f(x) = 3x^2 - 2x + 1$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(1,2)$.

We need two things:

i. A point on the line (We have that: $(x_1, y_1) = (1,2)$)

ii. The slope of the line (This is $f'(x_1)$)

$f'(x) = 6x - 2$

At the point $(x_1, y_1) = (1,2)$, **the slope is** $f'(2) = 6(1) - 2 = 4$

We will use the Point-Slope equation of a line:

$y - y_1 = m(x - x_1)$ (Where m is the slope and (x_1, y_1) is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$y - 2 = 4(x - 1)$

> The equation of the line tangent is $y - 2 = 4(x - 1)$

6. Given that $y = 4x^3 + 4x^2$ and that $x = \sec(t)$; compute $\frac{dy}{dt}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

We know:

$\frac{dy}{dx} = 12x^2 + 8x$

$\frac{dx}{dt} = \sec(t)\tan(t)$

We want: $\frac{dy}{dt}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (12x^2 + 8x)(\sec(t)\tan(t)) = \underbrace{\left(12(\sec(t))^2 + 8\sec(t)\right)\sec(t)\tan(t)}_{\text{express solely in terms of independent variable } t}$$

i.e. $\frac{dy}{dt} = \left(12(\sec(t))^2 + 8\sec(t)\right)\sec(t)\tan(t)$

7. Compute: $\frac{d}{dx}\left[\cos\left(5x^2 + 4x + 3\right)\right] =$

Outer:	=	$\cos(\quad)$
Deriv. of outer	=	$-\sin(\quad)$

$$\frac{d}{dx}\left[\begin{array}{l} \cos\left(\underbrace{5x^2 + 4x + 3}\right) \\ \uparrow \qquad\quad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{-\sin\left(5x^2 + 4x + 3\right)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{(10x + 4)}_{\text{deriv. of inner}}$$

i.e., $\frac{d}{dx}\left[\cos\left(5x^2 + 4x + 3\right)\right] = -\sin\left(5x^2 + 4x + 3\right)(10x + 4)$

8. Compute: $\frac{d}{dx}\left[\left(\frac{6x^2+9x}{2x^5+10x}\right)^4\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{6x^2+9x}{2x^5+10x}\right)^4}_{(g(x))^n}\right] = \underbrace{4\left(\frac{6x^2+9x}{2x^5+10x}\right)^3}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{6x^2+9x}{2x^5+10x}\right]\right)}_{\text{deriv of inner Function}}$$

$$= 4\left(\frac{6x^2+9x}{2x^5+10x}\right)^3 \underbrace{\frac{(12x+9)\left(2x^5+10x\right) - \left(10x^4+10\right)\left(6x^2+9x\right)}{\left(2x^5+10x\right)^2}}_{\text{quotient rule}}$$

$$\text{i.e., } \frac{d}{dx}\left[\left(\frac{6x^2+9x}{2x^5+10x}\right)^4\right] = 4\left(\frac{6x^2+9x}{2x^5+10x}\right)^3 \cdot \frac{(12x+9)\left(2x^5+10x\right) - \left(10x^4+10\right)\left(6x^2+9x\right)}{\left(2x^5+10x\right)^2}$$

9. Compute: $\frac{d}{dx}\left[\tan^{10}\left(2x^5+10x\right)\right] =$ Re-write!

$\frac{d}{dx}\left[\left(\tan\left(2x^5+10x\right)\right)^{10}\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[\left(\tan\left(2x^5+10x\right)\right)^{10}\right] = \underbrace{10\left(\tan\left(2x^5+10x\right)\right)^9}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\tan\left(2x^5+10x\right)\right]\right)}_{\text{derivative of inner}}$$

$$= 10\left(\tan\left(2x^5+10x\right)\right)^9 \cdot \underbrace{\left(\sec^2\left(2x^5+10x\right)\right)\cdot\left(10x^4+10\right)}_{\text{Chain Rule}}$$

$$\text{i.e., } \frac{d}{dx}\left[\tan^{10}\left(2x^5+10x\right)\right] = 10\left(\tan\left(2x^5+10x\right)\right)^9\left(\sec^2\left(2x^5+10x\right)\right)\cdot\left(10x^4+10\right)$$

10. Given that $S'(x) = \frac{1}{2S(x)}$; compute $\frac{d}{dx}[S(\sin(x))]$

Outer:	$=$	$S(\quad)$
Deriv. of outer	$=$	$\frac{1}{2S(\quad)}$

$$\frac{d}{dx}\Big[\underset{\uparrow\ \text{outer}}{S}\Big(\underbrace{\sin(x)}_{\uparrow\ \text{inner}}\Big)\Big] = \underbrace{\frac{1}{2S(\sin(x))}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\cos(x)}_{\text{deriv. of inner}} = \frac{\cos(x)}{2S(\sin(x))}$$

i.e., $\frac{d}{dx}[S(\sin(x))] = \frac{1}{2S(\sin(x))} \cdot \cos(x) = \frac{\cos(x)}{2S(\sin(x))}$

11. Given that $f(x) = 8x^2 - 2x + 3$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[8(x+\Delta x)^2 - 2(x+\Delta x)+3\right] - \left[8x^2-2x+3\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[8\left(x^2+2x\Delta x+\Delta x^2\right) - 2(x+\Delta x)+3\right] - \left[8x^2-2x+3\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[8x^2+16x\Delta x+8\Delta x^2-2x-2\Delta x+3\right] - \left[8x^2-2x+3\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{16x\Delta x+8\Delta x^2-2\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(16x+8\Delta x-2)}{\Delta x} = \lim_{\Delta x \to 0}(16x + 8\Delta x - 2) = 16x + 8(0) - 2 = 16x - 2$

i.e., $f'(x) = 16x - 2$