MTH 1125 Test #1 - (12 pm class) - Solutions

Fall 2023

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3}\frac{x^2+2x-8}{x^2+3x+5} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3}\frac{x^2+2x-8}{x^2+3x+5} = \frac{(3)^2+2(3)-8}{(3)^2+3(3)+5} = \frac{7}{23}$$

i.e., $\lim_{x\to 3}\frac{x^2+2x-8}{x^2+3x+5} = \frac{7}{23}$

2. Compute: $\lim_{x\to 3}\frac{x^2-x-6}{2x^2-5x-3} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3}\frac{x^2-x-6}{2x^2-5x-3} = \frac{(3)^2-(3)-6}{2(3)^2-5(3)-3} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3}\frac{x^2-x-6}{2x^2-5x-3} = \lim_{x\to 3}\frac{(x-3)(x+2)}{(x-3)(2x+1)} = \lim_{x\to 3}\frac{(x+2)}{(2x+1)} = \frac{(3)+2}{2(3)+1} = \frac{5}{7}$$

i.e., $\lim_{x\to 3}\frac{x^2-x-6}{2x^2-5x-3} = \frac{5}{7}$

3. Compute: $\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+2x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+2x-8} = \frac{(2)^2+2(2)-9}{(2)^2+2(2)-8} = \frac{-1}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 2^-} \frac{x^2+2x-9}{x^2+2x-8} = \lim_{x\to\ 2^-} \frac{x^2+2x-9}{(x+4)(x-2)} = \frac{-1}{(6)(-\varepsilon)} = \frac{1}{(6)(\varepsilon)} = \frac{\left(\frac{1}{6}\right)}{(\varepsilon)} = \infty$$

$$\begin{aligned} & x \to 2^- \\ \Rightarrow\ & x < 2 \\ \Rightarrow\ & x - 2 < 0 \end{aligned}$$

$$\lim_{x\to\ 2^+} \frac{x^2+2x-9}{x^2+2x-8} = \lim_{x\to\ 2^+} \frac{x^2+2x-9}{(x+4)(x-2)} = \frac{-1}{(6)(+\varepsilon)} = \frac{\left(-\frac{1}{6}\right)}{(\varepsilon)} = -\infty$$

$$\begin{aligned} & x \to 2^+ \\ \Rightarrow\ & x > 2 \\ \Rightarrow\ & x - 2 > 0 \end{aligned}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 2} \frac{x^2+2x-9}{x^2+2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{8x^6+5x-5}{4x^4+6x^3-8x} =$

$$\lim_{x\to\ -\infty} \frac{8x^6+5x-5}{4x^4+6x^3-8x} = \lim_{x\to\ -\infty} \frac{8x^6}{4x^4} = \lim_{x\to\ -\infty} 2x^2 = +\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{8x^6+5x-5}{4x^4+6x^3-8x} = +\infty$

5. $f(x) = \frac{2x^2-2x-3}{x^2+x-2}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow x = -2$ and $x = 1$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$\lim_{x\to\ -2^-} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to\ -2^-} \frac{2x^2-2x-3}{(x+2)(x-1)} = \frac{9}{(-\varepsilon)(-3)} = \frac{9}{(\varepsilon)(3)} = \frac{3}{\varepsilon} = +\infty$

$$\boxed{\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \end{array}}$$

$\lim_{x\to\ -2^+} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to\ -2^+} \frac{2x^2-2x-3}{(x+2)(x-1)} = \frac{9}{(+\varepsilon)(-3)} = \frac{\left(\frac{9}{-3}\right)}{\varepsilon} = \frac{-3}{\varepsilon} = -\infty$

$$\boxed{\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$\lim_{x\to\ 1^-} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to\ 1^-} \frac{2x^2-2x-3}{(x+2)(x-1)} = \frac{-3}{(3)(-\varepsilon)} = \frac{-1}{-\varepsilon} = +\infty$

$$\boxed{\begin{array}{ll} & x \to 1^- \\ \Rightarrow & x < 1 \end{array}}$$

$\lim_{x\to\ 1^+} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to 1^+} \frac{2x^2-2x-3}{(x+2)(x-1)} = \frac{-3}{(3)(\varepsilon)} = \frac{-1}{\varepsilon} = -\infty$

$$\boxed{\begin{array}{ll} & x \to 1^+ \\ \Rightarrow & x > 1 \end{array}}$$

Since the one-sided limits are **infinite,** $x = 1$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to-\infty} \frac{2x^2}{x^2} = \lim_{x\to-\infty} 2 = 2$

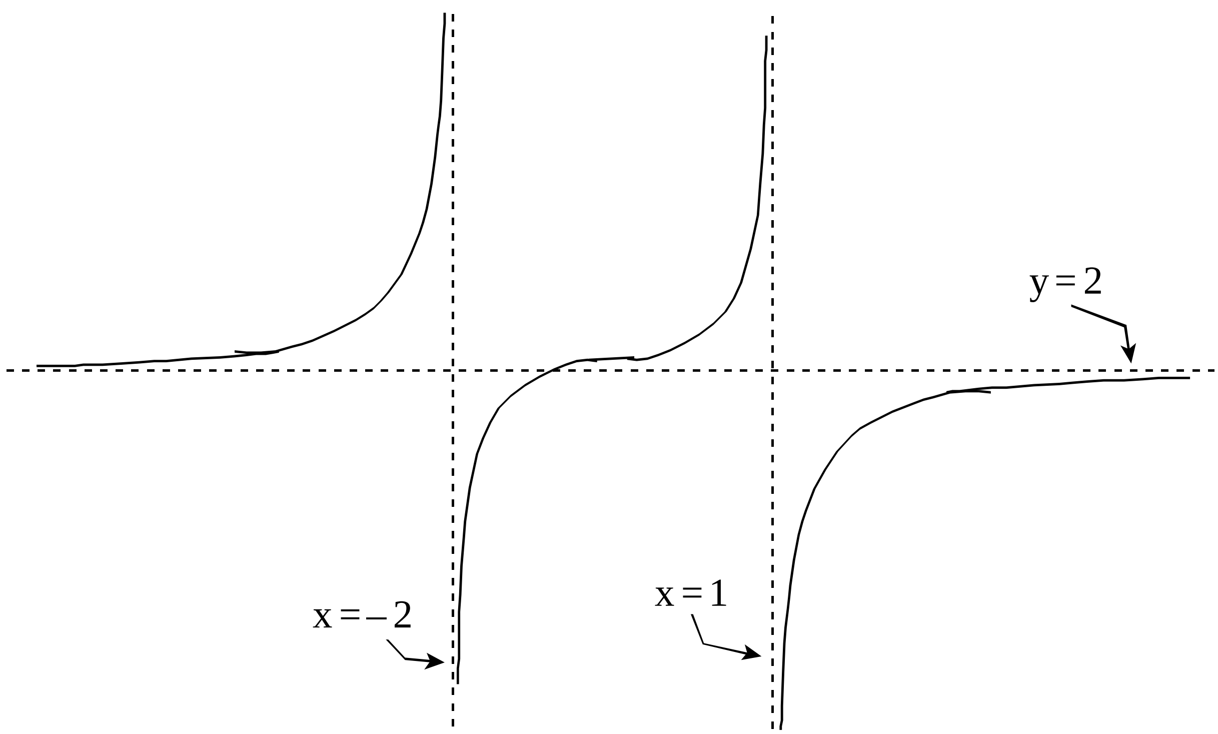
$\lim_{x\to+\infty} \frac{2x^2-2x-3}{x^2+x-2} = \lim_{x\to+\infty} \frac{2x^2}{x^2} = \lim_{x\to+\infty} 2 = 2$

Since the limits are **finite** and **constant,** $y = 2$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to\ -2^-} \frac{2x^2-2x-3}{x^2+x-2} = +\infty$	
$\lim_{x\to\ -2^+} \frac{2x^2-2x-3}{x^2+x-2} = -\infty$	$\lim_{x\to-\infty} \frac{2x^2-2x-3}{x^2+x-2} = 2$
$\lim_{x\to\ 1^-} \frac{2x^2-2x-3}{x^2+x-2} = +\infty$	$\lim_{x\to+\infty} \frac{2x^2-2x-3}{x^2+x-2} = 2$
$\lim_{x\to\ 1^+} \frac{2x^2-2x-3}{x^2+x-2} = -\infty$	

Graph $f(x) = \frac{2x^2-2x-3}{x^2+x-2}$



6. Compute: $\lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} =$

Step #1 Try Plugging in:

$$\lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} = \frac{\sqrt{(5)+11}-4}{(5)-5} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} = \lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} \cdot \frac{\sqrt{x+11}+4}{\sqrt{x+11}+4} = \lim_{x\to 5} \frac{\left(\sqrt{x+11}\right)^2-(4)^2}{(x-5)\left[\sqrt{x+11}+4\right]}$$

$$= \lim_{x\to 5} \frac{(x+11)-16}{(x-5)\left[\sqrt{x+11}+4\right]} = \lim_{x\to 5} \frac{(x-5)}{(x-5)\left[\sqrt{x+11}+4\right]} = \lim_{x\to 5} \frac{1}{\left[\sqrt{x+11}+4\right]}$$

$$= \frac{1}{\left[\sqrt{(5)+11}+4\right]} = \frac{1}{[4+4]} = \frac{1}{8}$$

i.e., $\lim_{x\to 5} \frac{\sqrt{x+11}-4}{x-5} = \frac{1}{8}$

7.

$x =$	$f(x) =$
1.5	-10
1.9	-100
1.99	$-1,000$
1.999	$-10,000$
1.9999	$-100,000$

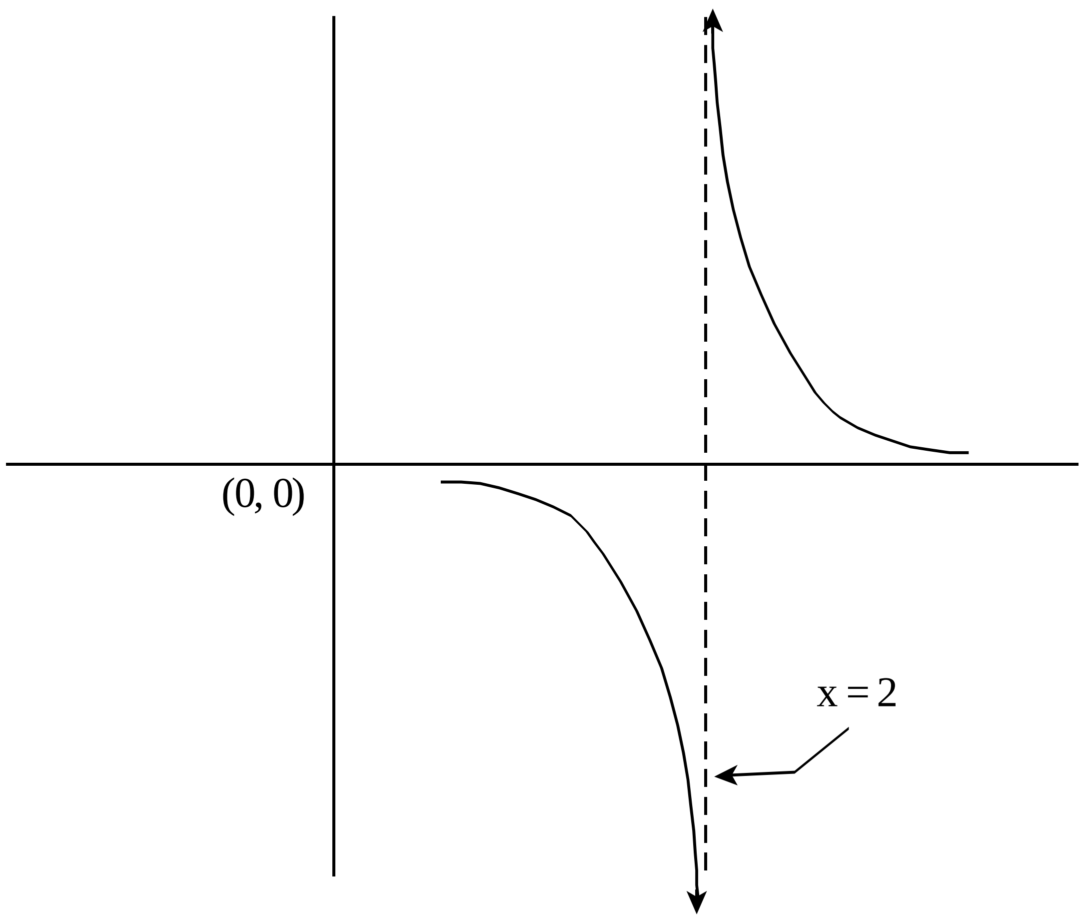
$x =$	$f(x) =$
2.5	10
2.1	100
2.01	$1,000$
2.001	$10,000$
2.0001	$100,000$

Based on the information in the table above, compute/do the following:

(a) $\lim_{x\to 2^-} f(x) = -\infty$

(b) $\lim_{x\to 2^+} f(x) = \infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 3$. (Justify Your Answer)

$$f(x) = \begin{cases} 2x+3 & \text{for } x < 3 \\ 9 & \text{for } x = 3 \\ 5x-6 & \text{for } x > 3 \end{cases}$$

$f(x)$ is continuous at the point $x = 3$ exactly when $\lim_{x\to 3} f(x) = f(3)$

Since the definition of $f(x)$ changes at $x = 3$, we must compute the one-sided limits as x approaches 3, in order to compute $\lim_{x\to 3} f(x)$.

Observe: As $x \to 3^-$, $x < 3$.

Therefore: $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (2x+3) = 2(3) + 3 = 9$

Also: As $x \to 3^+$, $x > 3$.

Therefore: $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (5x-6) = 5(3) - 6 = 9$

Since the one-sided limits are equal, $\lim_{x\to 3} f(x)$ exists, and is equal to the common value of the one-sided limits.

i.e., $\lim_{x\to 3} f(x) = 9$

Finally, note that $f(3) = 9$

$\Rightarrow \lim_{x\to 3} f(x) = f(3)$

Hence, $f(x)$ is continuous at the point $x = 3$