MTH 1125 12pm Class - Test #4 - Solutions

Fall 2023

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Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int (8x^3 + 12x^2 - 12x + 8 + 15\sqrt{x})\,dx =$

$$\int (8x^3 + 12x^2 - 12x + 8 + 15\sqrt{x})\,dx = \int \left(8x^3 + 12x^2 - 12x + 8 + 15x^{\frac{1}{2}}\right) dx$$

$$= 8\left[\tfrac{x^4}{4}\right] + 12\left[\tfrac{x^3}{3}\right] - 12\left[\tfrac{x^2}{2}\right] + 8x + 15\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + C = 2x^4 + 4x^3 - 6x^2 + 8x + 10x^{\frac{3}{2}} + C$$

i.e., $\int (8x^3 + 12x^2 - 12x + 8 + 15\sqrt{x})\,dx = 2x^4 + 4x^3 - 6x^2 + 8x + 10x^{\frac{3}{2}} + C$

2. **Compute:** $\int \left(9x^2+12x+5\right)^4 (3x+2)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(9x^2+12x+5\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (9x^2+12x+5)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(9x^2+12x+5)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+2)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (9x^2+12x+5)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 9x^2+12x+5 \\ \Rightarrow \tfrac{du}{dx} &= 18x+12 \\ \Rightarrow du &= (18x+12)\,dx \\ \Rightarrow \tfrac{1}{6}du &= (3x+2)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(9x^2+12x+5\right)^4}_{u^4}\underbrace{(3x+2)\,dx}_{\frac{1}{6}du} = \int u^4 \tfrac{1}{6}du = \tfrac{1}{6}\int u^4 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{6}\int u^4 du = \tfrac{1}{6}\left[\tfrac{u^5}{5}\right] + C = \tfrac{1}{30}u^5 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(9x^2+12x+5\right)^4 (3x+2)\,dx = \underbrace{\frac{1}{30}\left(9x^2+12x+5\right)^5 + C}_{\frac{1}{30}u^5+C}$$

i.e., $\int \left(9x^2+12x+5\right)^4(3x+2)\,dx = \frac{1}{30}\left(9x^2+12x+5\right)^5+C$

3. **Compute:** $\int (6\cos(x)+3\csc^2(x)+8\sec(x)\tan(x))\,dx =$

$\int (6\cos(x)+3\csc^2(x)+8\sec(x)\tan(x))\,dx = 6[\sin(x)]+3[-\cot(x)]+8[\sec(x)]+C$

$= 6\sin(x)-3\cot(x)+8\sec(x)+C$

i.e., $\int (6\cos(x)+3\csc^2(x)+8\sec(x)\tan(x))\,dx = 6\sin(x)-3\cot(x)+8\sec(x)+C$

4. **Compute:** $\int \cos\left(5x^2+12x+4\right)(5x+6)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(5x^2+12x+4\right)$

   $\nearrow$ outer $\nwarrow$ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(5x^2+12x+4\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(5x^2+12x+4\right)}_{\text{function}} - - - - \rightarrow \underbrace{(5x+6)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(5x^2+12x+4\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 5x^2+12x+4 \\ \Rightarrow \frac{du}{dx} &= 10x+12 \\ \Rightarrow du &= (10x+12)\,dx \\ \Rightarrow \tfrac{1}{2}du &= (5x+6)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(5x^2+12x+4\right)}_{\cos(u)}\underbrace{(5x+6)\,dx}_{\frac{1}{2}du} = \int \cos(u)\ \tfrac{1}{2}du = \tfrac{1}{2}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{2}\int \cos(u)\,du = \tfrac{1}{2}\left[\sin(u)\right] + C = \tfrac{1}{2}\sin(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \cos\left(5x^2+12x+4\right)(5x+6)\,dx = \underbrace{\frac{1}{2}\sin\left(5x^2+12x+4\right) + C}_{\frac{1}{2}\sin(u)+C}$$

i.e., $\int \cos\left(5x^2+12x+4\right)\left(5x+6\right)dx = \frac{1}{2}\sin\left(5x^2+12x+4\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(9x^2+2x+4\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(9x^2+2x+4\right)}_{f(x)}dx = \underbrace{\left[9\left(\frac{x^3}{3}\right)+2\left(\frac{x2}{2}\right)+4x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[3x^3+x^2+4x\right]_{-1}^{1}}$$

$$= \underbrace{\left[3\left(1\right)^3+\left(1\right)^2+4\left(1\right)\right]}_{F(1)} - \underbrace{\left[3\left(-1\right)^3+\left(-1\right)^2+4\left(-1\right)\right]}_{F(-1)}$$

$$= 8-(-6) = 14$$

i.e., $\int_{-1}^{1}\left(9x^2+2x+4\right)dx = 14$

6. **Compute:** $\int_0^1 \left(x^2-2x+2\right)^3(x-1)\,dx =$ (The answer may not be a whole number or a positive number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(x^2-2x+2\right)^3$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (x^2-2x+2)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(x^2-2x+2\right)}_{\text{function}} - - - - \rightarrow \underbrace{(x-1)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (x^2-2x+2)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= (x^2-2x+2) \\ \Rightarrow \frac{du}{dx} &= 2x-2 \\ \Rightarrow du &= (2x-2)\ dx \\ \Rightarrow \tfrac{1}{2}du &= (x-1)\ dx \end{aligned}$$

When $x=0$, $u = x^2-2x+2 = (0)^2-2(0)+2 = 2$
When $x=1$, $u = x^2-2x+2 = (1)^2-2(1)+2 = 1$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{\left(x^2-2x+2\right)^3}_{u^3} \underbrace{(x-1)\ dx}_{\frac{1}{2}du} = \int_{u=2}^{u=1} u^3 \cdot \tfrac{1}{2}du = \tfrac{1}{2}\int_{u=2}^{u=1} u^3 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{2}\int_{u=2}^{u=1} u^3 du = \frac{1}{2}\left[\frac{u^4}{4}\right]_{u=2}^{u=1} = \left[\frac{u^4}{8}\right]_{u=2}^{u=1} = \underbrace{\frac{(1)^4}{8}}_{F(1)} - \underbrace{\frac{(2)^4}{8}}_{F(2)} = \frac{1}{8} - \frac{16}{8} = -\frac{15}{8}$$

i.e., $\int_0^1 (x^2 - 2x + 2)^3 (x - 1)\ dx = -\frac{15}{8}$