Integration By Parts - Special Problems

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Instructions. When performing integration by parts on an integral which contains a transcendental function whose derivative is algebraic (e.g. $\tan^{-1}(x)$ has, as its derivative, $\frac{1}{1+x^2}$), the course of action is to let u be the transcendental function. For example, given $\int x\tan^{-1}(x)\,dx$, we would let $u = \tan^{-1}(x)$, and we would re-write the integrand so that $u = \tan^{-1}(x)$ would come first, giving us: $\int \underbrace{\tan^{-1}(x)}_{u}\underbrace{xdx.}_{dv}$

Compute the following, with this observation in mind.

1. $\int x\tan^{-1}(x)\,dx =$

   Answer: $\frac{1}{2}x^2\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C$

2. $\int x\ln(x)\,dx =$

   Answer: $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

3. $\int \sin^{-1}(x)\,xdx =$

   Answer: $\frac{1}{2}x^2\arcsin x + \frac{1}{4}x\sqrt{(1-x^2)} - \frac{1}{4}\arcsin x + C$

4. $\int x^3\tan^{-1}(x)\,dx =$

   Answer: $\frac{1}{4}x^4\arctan x - \frac{1}{12}x^3 + \frac{1}{4}x - \frac{1}{4}\arctan x + C$

5. $\int \tan^{-1}(x)\,dx =$

   Answer: $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

6. $\int \ln(x)\,dx =$

   Answer: $x\ln x - x + C$

7. $\int \sin^{-1}(x)\,dx =$

   Answer: $x\arcsin(x) + \sqrt{(1-x^2)} + C$