

MTH 3318 Induction Set 1b

FALL 2022

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Name _____

Instructions. Prove the following by mathematical induction.

1. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that:
 $|x_1 + x_2 + x_3 + \dots + x_n| \leq |x_1| + |x_2| + |x_3| + \dots + |x_n|$ for real numbers x_1, x_2, \dots, x_n ,
(the General Triangle Inequality).
2. $(1 + x)^n \geq 1 + nx$ for any natural number n and any real number $x \geq -1$.
3. For real numbers a, b , with $0 \leq a \leq b$; prove that $a^n \leq b^n$.
4. $n(n + 1)$ is divisible by 2 for all natural numbers, n .
5. $n(n + 1)(n + 2)$ is divisible by 6 for all natural numbers, n .
6. Given a set A with n elements, the **power set of A**, denoted 2^A , is the collection of all subsets of A .

For example, given the set $A = \emptyset$, the power set of A is

$$2^A = \{\emptyset\} \quad (2^A \text{ has } 2^0 = 1 \text{ element})$$

For example, given the set $A = \{1\}$, the power set of A is

$$2^A = \{\emptyset, \{1\}\} \quad (2^A \text{ has } 2^1 = 2 \text{ elements})$$

For example, given the set $A = \{1, 2\}$, the power set of A is

$$2^A = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\} \quad (2^A \text{ has } 2^2 = 4 \text{ elements})$$

For example, given the set $A = \{1, 2, 3\}$, the power set of A is

$$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\} \quad (2^A \text{ has } 2^3 = 8 \text{ elements})$$

Prove the following: If A is a set with n elements, then the power set of A has 2^n elements. (Hint: For the induction step, don't consider two unrelated sets - one with n elements, and the other one with $n + 1$ elements. Instead, form a set with $n + 1$ elements by taking a set with n elements and adding one new element to it.)