

# MTH 4441 Test #2

FALL 2023

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Name \_\_\_\_\_

1. Define - Cyclic group

2. Define - Direct Product of Groups  $(G, *G)$  and  $(H *H)$

3. Define - Isomorphism

4. **Prove or Disprove:**  $(\mathbb{R}, +)$  is a cyclic group

5. **Prove or Disprove:**  $(\mathbb{Q}, +)$  is a cyclic group

6. Compute the sum of the elements  $(5, 2)$  and  $(4, 2)$  in the group  $\mathbb{Z}_7 \times \mathbb{Z}_4$

7. Given the group table for  $(G, *)$ , find all of the subgroups of  $(G, *)$  and justify your answers. Draw a subgroup diagram for  $(G, *)$ .

$*$	$e$	$a$	$b$	$c$	$d$
$e$	$e$	$a$	$b$	$c$	$d$
$a$	$a$	$b$	$c$	$d$	$e$
$b$	$b$	$c$	$d$	$e$	$a$
$c$	$c$	$d$	$e$	$a$	$b$
$d$	$d$	$e$	$a$	$b$	$c$

8. Construct the group table for  $(U_7, \odot)$ , and then find all of the subgroups of  $(U_7, \odot)$  and justify your answers. Draw a subgroup diagram for  $(U_7, \odot)$ . (Recall:  $U_7 = \{1, 2, 3, 4, 5, 6\}$ )

9. Construct the group table for  $(\mathbb{Z}_6, \oplus)$ , and then find all of the subgroups of  $(\mathbb{Z}_6, \oplus)$  and justify your answers. Draw a subgroup diagram for  $(\mathbb{Z}_6, \oplus)$ .
10. Construct the group table for  $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$ , and then find all of the subgroups of  $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$  and justify your answers. Draw a subgroup diagram for  $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$ .
11. With reference to Exercises 9 and 10, Define an isomorphism from  $(\mathbb{Z}_6, \oplus)$  to  $(\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$  and prove that  $(\mathbb{Z}_6, \oplus) \cong (\mathbb{Z}_3 \times \mathbb{Z}_2, \oplus)$

12. (Extra - 5 pts) Calculate the order of the element  $(4, 9)$  in the group  $\mathbb{Z}_{18} \times \mathbb{Z}_{12}$

13. (Extra - 5 pts) Calculate the order of the element  $(8, 6, 4)$  in the group  $\mathbb{Z}_{18} \times \mathbb{Z}_9 \times \mathbb{Z}_8$