

Exercises Involving Real Numbers #5

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Pat Rossi

Name _____

Instructions. Prove the following:by contradiction.

1. If n^2 is even, then so is n .
2. If n^2 is odd, then so is n .
3. If n^r is odd, then so is n . (Here, r is a natural number.)
4. If n^r is even, then so is n . (Here, r is a natural number.) (This can be done using the result of #3)
 - (a) Alternatively, this can be done as a proof using contradiction, along with an induction argument.
5. If $\frac{n^2}{r^2} = p$, where p is a prime number, then n is divisible by p .
6. If n is divisible by r , then n^2 is divisible by r^2 . (Here, r is a natural number.)
7. If n is not divisible by r , then n^p is also not divisible by r . (Here, p and r are natural numbers.)
8. $\frac{n}{n+a} \leq \frac{n+a}{n+2a}$
9. $\frac{n^2}{n^2+1} \leq \frac{n^2+1}{n^2+2}$
10. $a + b \geq \sqrt{a^2 + b^2}$ (for all non-negative real numbers, a and b)
11. $\sqrt{ab} \leq \frac{(a+b)}{2}$ (for all non-negative real numbers, a and b)
12. If $x + y \geq 2$, then either $x \geq 1$ or $y \geq 1$.
13. Show that if $xy = 0$, then either $x = 0$ or $y = 0$. (You can assume that if a, b , and c are real numbers with $ab = ac$ and $a \neq 0$, then $b = c$.)
14. Show that if 100 balls are placed into 9 boxes, then some box contains 12 or more balls.