MTH 3318 - Test #2

Spring 2024

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Instructions. Fully document your work.

1. In exercises 1.a - 1.d, let p be the statement: "He pays me," and let q be the statement: "I will do the work." Write each statement in symbolic form.

   (a) If he pays me, then I will do the work.

   If $\underbrace{\text{he pays me,}}_{p}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will do the work.}}_{q}$

   $\boxed{p \rightarrow q}$

   (b) He will pay me, or I will not do the work.

   $\underbrace{\text{He will pay me}}_{p}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{I will not do the work.}}_{\sim q}$

   $\boxed{p \vee (\sim q)}$

   (c) His paying me is a necessary and sufficient condition for me to do the work.

   $\underbrace{\text{His paying me}}_{p}$ $\underbrace{\text{is a necessary and sufficient condition for}}_{\leftrightarrow}$ $\underbrace{\text{me to do the work.}}_{q}$

   $\boxed{p \leftrightarrow q}$

   (d) He will pay me if I do the work.

   $\underbrace{\text{He will pay me}}_{p}$ $\underbrace{\text{if}}_{\leftarrow}$ $\underbrace{\text{I do the work.}}_{q}$

   $\boxed{p \leftarrow q \quad \text{or} \quad q \rightarrow p}$

2. In exercises 2.a - 2.d, let p be the statement: "I will buy new clothes," and let q be the statement: "I will look good." Write each statement in words.

   (a) $p \wedge q$

   $$\underbrace{\text{I will buy new clothes}}_{p}\ \underbrace{\text{and}}_{\wedge}\ \underbrace{\text{I will look good.}}_{q}$$

   (b) $p \vee q$

   $$\underbrace{\text{I will buy new clothes}}_{p}\ \underbrace{\text{or}}_{\vee}\ \underbrace{\text{I will look good.}}_{q}$$

   (c) $q \rightarrow \sim p$

   $$\text{If }\underbrace{\text{I look good,}}_{q}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{I will not buy new clothes.}}_{\sim p}$$

   (d) $\sim p \leftrightarrow \sim q$

   $$\underbrace{\text{I will not buy new clothes}}_{\sim p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{I do not look good.}}_{\sim q}$$

3. In problems 3.a - 3.d, determine whether the given propositions are True or False:

   (a) $\text{If }\underbrace{8+3=9,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>10}_{F}.$

   $\boxed{F \rightarrow F = T}$

   (b) $\underbrace{\text{If } 8>3,}_{T}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>5.}_{T}$

   $\boxed{T \rightarrow T = T}$

   (c) $\underbrace{8>10,}_{F}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{2+2=5.}_{F}$

   $\boxed{F \leftrightarrow F = T}$

   (d) $\text{If }\underbrace{2+2=5,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8>10.}_{F}$

   $\boxed{F \rightarrow F = T}$

4. In exercises 4.a-4.b construct a truth table for the statement given.

(a) $p \wedge (q \longleftrightarrow r)$

p	q	r	$(q \longleftrightarrow r)$	$p \wedge (q \longleftrightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	T	F

(b) $(\sim p \wedge q) \rightarrow \sim r$

p	q	r	$\sim p$	$\sim r$	$(\sim p \wedge q)$	$(\sim p \wedge q) \rightarrow \sim r$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

5. For problems 5.a - 5.d, negate the given statements:

   (a) All kids have freckles.

   Some kids don't have freckles.

   At least one kid doesn't have freckles.

   (b) No men snore.

   Some men snore.

   At least one man snores.

   (c) Some flowers have nectar.

   No flowers have nectar.

   (d) $\forall$ real numbers x, $\exists$ real number y, $\ni y = \frac{1}{x}$.
   (i.e. For all real numbers x, there exists a real number y such that $y = \frac{1}{x}$.)

   $\sim$ ($\forall$ real numbers x, $\exists$ real number y, $\ni y = \frac{1}{x}$.)

   $\Leftrightarrow \exists$ a real number x, $\ni \sim$ ($\exists$ real number y, $\ni y = \frac{1}{x}$.)

   $\Leftrightarrow \exists$ a real number x, $\ni \forall$ real numbers y, $\sim \left(y = \frac{1}{x}.\right)$

   $\Leftrightarrow \exists$ a real number x, $\ni \forall$ real numbers y, $\ni y \neq \frac{1}{x}$.

6. For problems 6.a - 6.b, disprove the given statements by providing a suitable counter-example:

   (a) $\forall n \in \mathbb{N}$, if $2n$ is even, then n is also even.

   Counter-example:

   Let $n = 3$.

   Then $2n = 6$ is even, but n is odd.

   (b) If x is a factor of $(y + z)$, then x is a factor of y and x is a factor of z.

   Counter-example:

   Let $x = 2$, $y = 3$, and $z = 5$.

   Then x is a factor of $(y + z)$, but x is neither a factor of y or z.

7. Write the converse, inverse, and contrapositive of the following statement, labeling each one.

   If $\underbrace{\text{I turn the key}}_{p}$, $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{the car will start.}}_{q}$

   Converse: If $\underbrace{\text{the car will start,}}_{q}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will turn the key}}_{p}$

   Inverse: If $\underbrace{\text{I don't turn the key}}_{\sim p}$, $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{the car will not start.}}_{\sim q}$

   Contrapositive: If $\underbrace{\text{the car will not start,}}_{\sim q}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will not turn the key}}_{\sim p}$

8. In problems 8.a - 8.b, determine whether the given arguments are valid.

(a) I will make him a partner if and only if he closes this deal. If he leaves tonight, then he will close this deal. Therefore, I will make him a partner if he leaves tonight.

The premises and the conclusion are composed of the following simple statements:

p : I will make him a partner

q : He closes this deal

r : He leaves tonight

Our premises and conclusion are:

p_1 : $\underbrace{\text{I will make him a partner}}_{p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{he closes this deal.}}_{q}$

p_2 : $\underbrace{\text{If he leaves tonight,}}_{r}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{\text{he will close this deal.}}_{q}$

c : $\underbrace{\text{I will make him a partner}}_{p}\ \underbrace{\text{if}}_{\leftarrow}\ \underbrace{\text{he leaves tonight.}}_{r}$

Our argument is of the form: (conjunction of the premises) $\rightarrow$ (conclusion)

p	q	r	p_1: $p \leftrightarrow q$	p_2: $r \rightarrow q$	$p_1 \wedge p_2$	c: $r \rightarrow p$	$(p_1 \wedge p_2) \rightarrow c$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	T	T
F	T	T	F	T	F	F	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Since the argument is a tautology, the argument is VALID

(b) Some dolphins are fish. All fish taste good. Therefore, some dolphins taste good.

p_1: Some dolphins are fish.

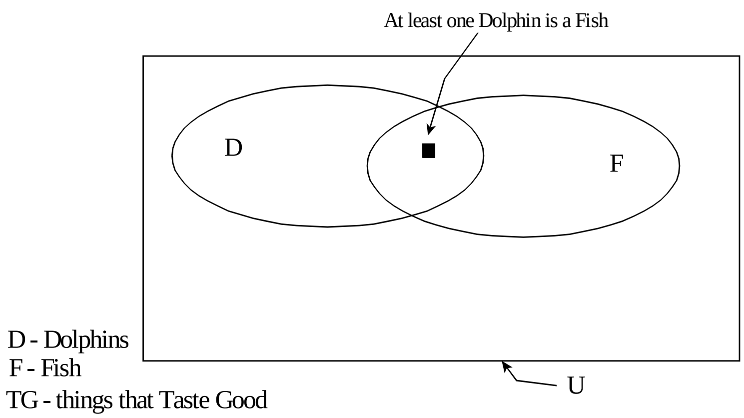
p_2: All fish taste good.

c : Some dolphins taste good.

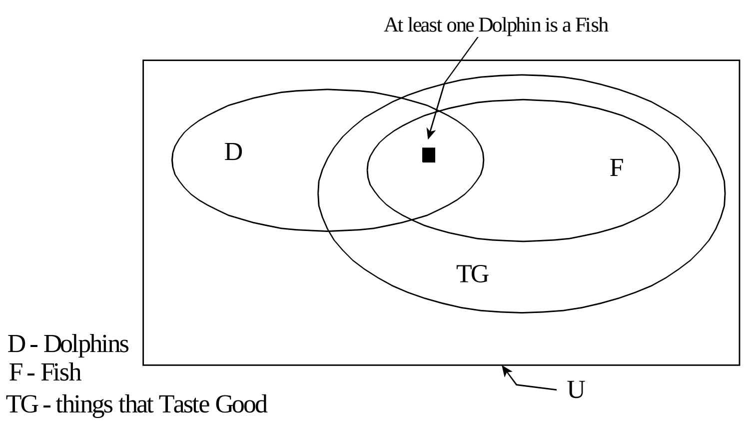
This is an argument that involves quantifiers. ("Some are," "All do," "Some . . . do"). Therefore, we determine its validity using Euler Circles.

We will draw the circles in such a way that the premises are true, but we will try to do this in such a way that the conclusion is false.

p_1: Some dolphins are fish.



Include the premise: p_2: All fish taste good.



Notice that the premise p_2 can't be drawn without making the conclusion true (The TG and D circles intersect).

Hence, the argument is VALID.

9. In problems 9.a - 9.b, determine whether the given arguments are valid.

(a) If I eat right and I exercise, then I will make the team. I will make the team. Therefore, if I don't make the team, then I don't eat right.

The premises and the conclusion are composed of the following simple statements:

p : I eat right

q : I exercise

r : I will make the team.

Our premises and conclusion are:

p_1: If $\underbrace{\text{I eat right and I exercise,}}_{p \wedge q}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will make the team.}}_{r}$

p_2: $\underbrace{\text{I will make the team.}}_{r}$

c : if $\underbrace{\text{I don't make the team,}}_{\sim r}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I don't eat right.}}_{\sim p}$

Our argument is of the form: (conjunction of the premises) $\rightarrow$ (conclusion)

p	q	r	$\sim p$	$\sim r$	$p \wedge q$	p_1: $(p \wedge q) \rightarrow r$	p_2: r	$p_1 \wedge p_2$	c: $(\sim r) \rightarrow (\sim p)$	$(p_1 \wedge p_2) \rightarrow c$
T	T	T	F	F	T	T	T	T	T	T
T	T	F	F	T	T	F	F	F	F	T
T	F	T	F	F	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F	F	T
F	T	T	T	F	F	T	T	T	T	T
F	T	F	T	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T	T	T	T
F	F	F	T	T	F	T	F	F	T	T

Since the argument is a tautology, the argument is VALID

(b) No squares are rectangles. Some triangles are squares. Therefore, some triangles are not rectangles.

	P_1:	No squares are rectangles.
	P_2:	Some triangles are squares.
$\therefore$	C:	Some triangles are not rectangles

This is an argument that involves quantifiers. ("Some are," "All do," "Some . . . do"). Therefore, we determine its validity using Euler Circles.

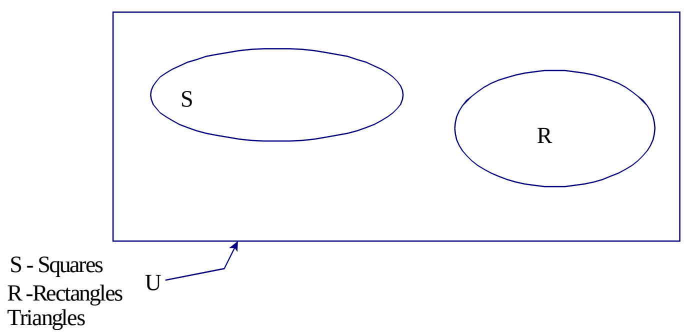
We will draw the circles in such a way that the premises are true, but we will try to do this in such a way that the conclusion is false.

We use the following notation:

S - Squares
R - Rectangles
T - Triangles

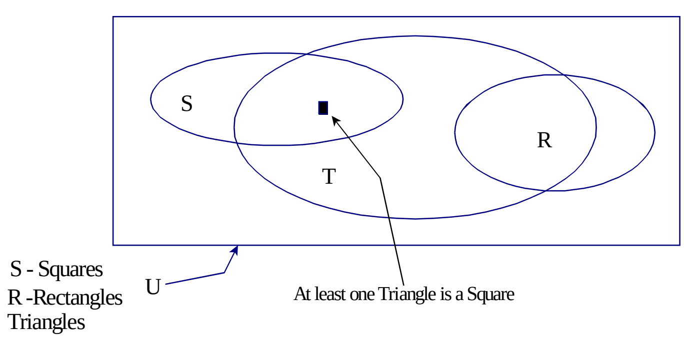
We will draw the circles in such a way that the premises are true, but we will try to do this in such a way that the conclusion is false.

p_1: No squares are rectangles.



Include the premise: p_2: Some triangles are squares.

Try to do this in such a way that the conclusion "Some triangles are not rectangles" is false.



Since the conclusion can be depicted as being False, while the premises are True, the argument is invalid