MTH 1125 Test #1 - (2 pm class) - Solutions

Fall 2022

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3}\frac{x^2-4x+8}{x^2-2x+5}=$

Step #1 Try Plugging In:

$\lim_{x\to 3}\frac{x^2-4x+8}{x^2-2x+5}=\frac{(3)^2-4(3)+8}{(3)^2-2(3)+5}=\frac{5}{8}$

i.e., $\lim_{x\to 3}\frac{x^2-4x+8}{x^2-2x+5}=\frac{5}{8}$

2. Compute: $\lim_{x\to 3}\frac{x^2-8x+15}{2x^2-7x+3}=$

$\lim_{x\to 3}\frac{x^2-8x+15}{2x^2-7x+3}=\frac{(3)^2-8(3)+15}{2(3)^2-7(3)+3}=\frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 3}\frac{x^2-8x+15}{2x^2-7x+3}=\lim_{x\to 3}\frac{(x-3)(x-5)}{(2x-1)(x-3)}=\lim_{x\to 3}\frac{x-5}{2x-1}=\frac{(3)-5}{2(3)-1}=\frac{-2}{5}=-\frac{2}{5}$

i.e., $\lim_{x\to 3}\frac{x^2-8x+15}{2x^2-7x+3}=-\frac{2}{5}$

3. Compute: $\lim_{x\to\ 4} \frac{x^2-2x-9}{x^2-2x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 4} \frac{x^2-2x-9}{x^2-2x-8} = \frac{(-4)^2-2(4)-9}{(-4)^2-2(4)-8} = \frac{-1}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 4^-} \frac{x^2-2x-9}{x^2-2x-8} = \lim_{x\to\ 4^-} \frac{x^2-2x-9}{(x+2)(x-4)} = \frac{-1}{(6)(-\varepsilon)} = \frac{1}{(6)(\varepsilon)} = \frac{\left(\frac{1}{6}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{ll} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \end{array}$$

$$\lim_{x\to\ 4^+} \frac{x^2-2x-9}{x^2-2x-8} = \lim_{x\to\ 4^+} \frac{x^2-2x-9}{(x+2)(x-4)} = \frac{-1}{(6)(\varepsilon)} = \frac{\left(\frac{-1}{6}\right)}{(+\varepsilon)} = \frac{\left(-\frac{1}{6}\right)}{+\varepsilon} = -\infty$$

$$\begin{array}{ll} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 4} \frac{x^2-2x-9}{x^2-2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{4x^5+6x^3-8x}{9x^4+7x-5} =$

$$\lim_{x\to\ -\infty} \frac{4x^5+6x^3-8x}{9x^4+7x-5} = \lim_{x\to\ -\infty} \frac{4x^5}{9x^4} = \lim_{x\to\ -\infty} \frac{4x}{9} = -\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{4x^5+6x^3-8x}{9x^4+7x-5} = -\infty$

5. $f(x) = \frac{x^2-4x-3}{x^2-8x+16} = \frac{x^2-4x-3}{(x-4)^2}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow (x-4)^2 = 0$

$\Rightarrow (x-4) = 0$

$\Rightarrow x = 4$ is a possible vertical asymptote.

2. Compute the one-sided limits.

$\lim_{x\to\ 4^-} \frac{x^2-4x-3}{(x-4)^2} = \lim_{x\to\ -4^-} \frac{-3}{(-\varepsilon)^2} = \frac{-3}{(\varepsilon)^2} = -\infty$

$$\boxed{\begin{array}{ll} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \end{array}}$$

$\lim_{x\to\ 4^+} \frac{x^2-4x-3}{(x-4)^2} = \lim_{x\to\ -4^+} \frac{-3}{(+\varepsilon)^2} = \frac{-3}{(\varepsilon)^2} = -\infty$

$$\boxed{\begin{array}{ll} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \end{array}}$$

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-4x-3}{(x-4)^2} = \lim_{x\to-\infty} \frac{x^2-4x-3}{x^2-8x+16} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

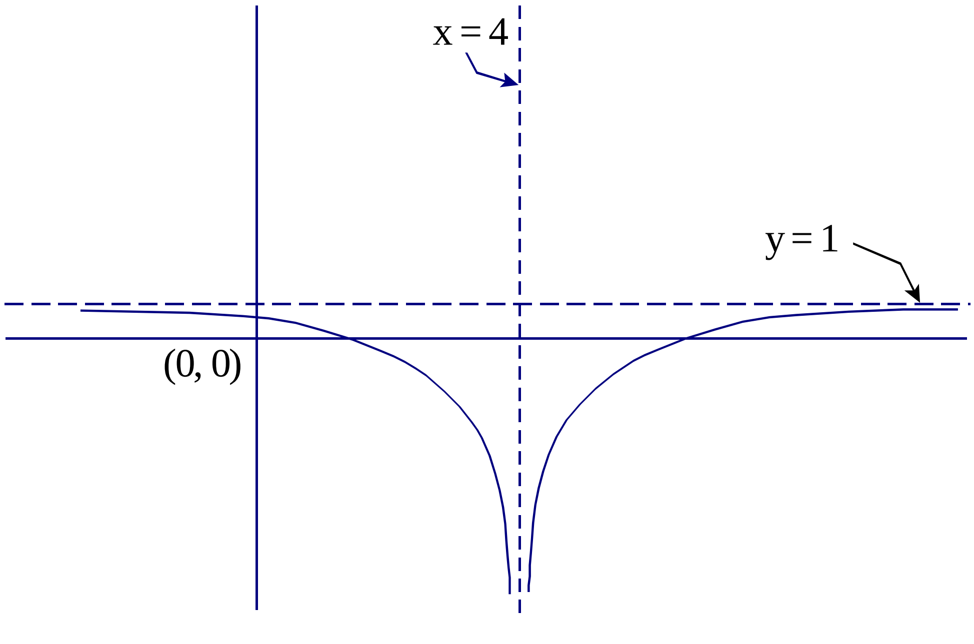
$\lim_{x\to+\infty} \frac{x^2-4x-3}{(x-4)^2} = \lim_{x\to+\infty} \frac{x^2-4x-3}{x^2-8x+16} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to 4^-} \frac{x^2-4x-3}{(x-4)^2} = -\infty$	$\lim_{x\to -\infty} \frac{x^2-4x-3}{(x-4)^2} = 1$
$\lim_{x\to 4^+} \frac{x^2-4x-3}{(x-4)^2} = -\infty$	$\lim_{x\to +\infty} \frac{x^2-4x-3}{(x-4)^2} = 1$

Graph $f(x) = \frac{x^2-4x-3}{x^2-8x+16} = \frac{x^2-4x-3}{(x-4)^2}$



6. Compute: $\lim_{x\to 10} \frac{\sqrt{x-1}-3}{x-10} =$

Step #1 Try Plugging in:

$$\lim_{x\to 10} \frac{\sqrt{x+1}-3}{x-10} = \lim_{x\to 10} \frac{\sqrt{(10)-1}-3}{(10)-10} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 10} \frac{\sqrt{x-1}-3}{x-10} = \lim_{x\to 10} \frac{\sqrt{x-1}-3}{x-10} \cdot \frac{\sqrt{x-1}+3}{\sqrt{x-1}+3} = \lim_{x\to 10} \frac{\left(\sqrt{x-1}\right)^2-(3)^2}{(x-10)\left[\sqrt{x-1}+3\right]}$$

$$= \lim_{x\to 10} \frac{(x-1)-9}{(x-10)\left[\sqrt{x-1}+3\right]} = \lim_{x\to 10} \frac{(x-10)}{(x-10)\left[\sqrt{x-1}+3\right]} = \lim_{x\to 10} \frac{1}{\left[\sqrt{x-1}+3\right]}$$

$$= \frac{1}{\left[\sqrt{(10)-1}+3\right]} = \frac{1}{3+3} = \frac{1}{6}$$

i.e., $\lim_{x\to 10} \frac{\sqrt{x-1}-3}{x-10} = \frac{1}{6}$

7.

$x =$	$f(x) =$
1.5	-10
1.9	-100
1.99	$-1,000$
1.999	$-10,000$
1.9999	$-100,000$

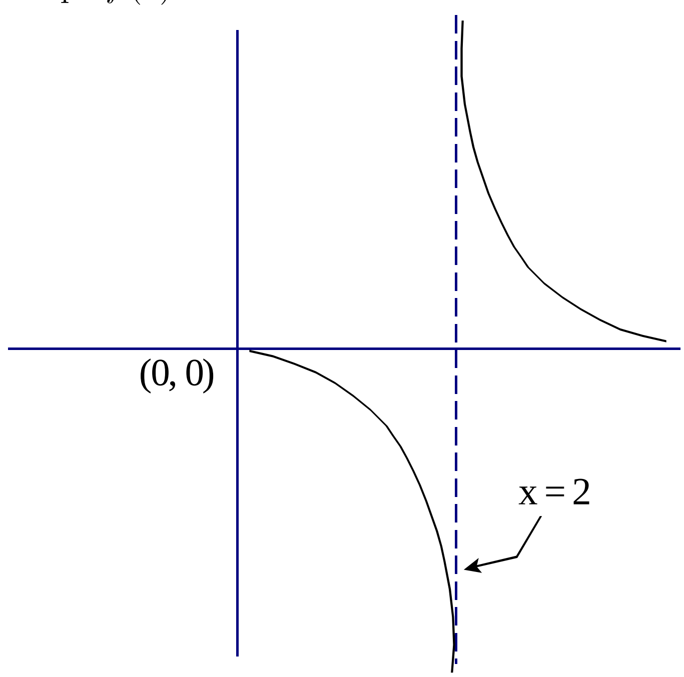
$x =$	$f(x) =$
2.5	10
2.1	100
2.01	$1,000$
2.001	$10,000$
2.0001	$100,000$

Based on the information in the table above, compute/do the following:

(a) $\lim_{x \to 2^-} f(x) = -\infty$

(b) $\lim_{x \to 2^+} f(x) = +\infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 4$. (Justify Your Answer)

$$f(x) = \begin{cases} 4x - 4 & \text{for } x < 4 \\ 12 & \text{for } x = 4 \\ x^2 - 4 & \text{for } x > 4 \end{cases}$$

First of all, let's recognize that $f(x)$ will be continuous at the point $x = 4$ exactly when $\lim_{x \to 4} f(x) = f(4)$.

So we should compute: $\lim_{x \to 4} f(x)$

The problem is that $f(x)$ is defined differently for $x < 4$ than it is for $x > 4$.

So we must compute the one sided limits as $x \to 4$

Observe: As $x \to 4^-$, $x < 4$.

Therefore: $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (4x - 4) = 4(4) - 4 = 12$

Also: As $x \to 4^+$, $x > 4$.

Therefore: $\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (x^2 - 4) = (4)^2 - 4 = 12$

Since the one-sided limits are equal, $\lim_{x \to 4} f(x)$ exists, and is equal to the common value of the one-sided limits.

i.e., $\lim_{x \to 4} f(x) = 12$

Finally, note that $f(4) = 12$

$\Rightarrow \lim_{x \to 4} f(x) = f(4)$

Hence, $f(x)$ is continuous at the point $x = 4$