Proofs Involving Sets #1 - Solutions

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Instructions. Prove the following.

1. $(A \setminus B) \subseteq (A \cup B)$

   Proof. Let $x \in (A \setminus B)$

   $\Rightarrow x \in A$ and $x \notin B$

   In particular, $x \in A$

   $\Rightarrow x \in A$ or $x \in B$

   $\Rightarrow x \in (A \cup B)$

   i.e., $x \in (A \setminus B) \Rightarrow x \in (A \cup B)$

   Hence, $(A \setminus B) \subseteq (A \cup B)$ ■

2. $A \cap B = A \Rightarrow A \subseteq B$

   Proof. Let the hypothesis be given. (i.e., let $A \cap B = A$).

   We need to show that $A \subseteq B$.

   So let $x \in A$

   $\Rightarrow x \in A \cap B$ (because $A = A \cap B$ by hypothesis).

   $\Rightarrow x \in A$ and $x \in B$

   In particular, $x \in B$.

   We have just shown that $x \in A \Rightarrow x \in B$.

   Hence, $A \subseteq B$ ■

3. $(A \cup B) = B \Rightarrow A \subseteq B$

   Proof. Let the hypothesis be given. (i.e., let $(A \cup B) = B$).

   We need to show that $A \subseteq B$

   So let $x \in A$

   $\Rightarrow x \in A$ or $x \in B$

   $\Rightarrow x \in (A \cup B)$

   $\Rightarrow x \in B$ (because $(A \cup B) = B$ by hypothesis).

   We have shown that $x \in A \Rightarrow x \in B$.

   Hence, $A \subseteq B$ ■

4. $A \subseteq B \Rightarrow B^c \subseteq A^c$

   Proof. Let the hypothesis be given. (i.e., let $A \subseteq B$).

   We need to show that $B^c \subseteq A^c$.

   So let $x \in B^c$

   $\Rightarrow x \notin B$

   $\Rightarrow x \notin A$ (Otherwise, if x *were* an element of A, then our hypothesis would imply that $x \in B$, contradicting the fact that $x \notin B$.)

   $\Rightarrow x \in A^c$.

   We have shown that $x \in B^c \Rightarrow x \in A^c$.

   Hence, $B^c \subseteq A^c$. ■

5. $(A \cap B) \subseteq A$

   Proof. Let $x \in (A \cap B)$

   $\Rightarrow x \in A$ and $x \in B$

   in particular, $x \in A$

   i.e., $x \in (A \cap B) \Rightarrow x \in A$

   Hence, $(A \cap B) \subseteq A$ ■

6. $A \subseteq (A \cup B)$

Proof. Let $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$

i.e., $x \in A \Rightarrow x \in (A \cup B)$

Hence, $A \subseteq (A \cup B)$ ■