

MTH 3318 Test #1

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Name _____

Instructions. Fully document your work.

For problems 1 - 2 prove one using Mathematical Induction.

1. $1 + 3 + 5 + \dots + (2n - 1) = n^2$

i.e. $\sum_{i=1}^n (2i - 1) = n^2$

2. $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

i.e. $\sum_{j=1}^n \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$

For problems 3 - 5 prove one using Mathematical Induction.

3. $1 + 5 + 9 + \dots + (4n - 3) = 2n^2 - n$

i.e., $\sum_{i=1}^n (4i - 3) = 2n^2 - n$

4. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

i.e. $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

5. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \dots + n^3$ all natural numbers, n .

For problems 6 - 7, prove one using Mathematical Induction:

6. $n(n + 1)$ is divisible by 2 for all natural numbers, n .

7. Given that $\frac{d}{dx} [x^0] = 0$ and $\frac{d}{dx} [x^1] = 1$, prove that $\frac{d}{dx} [x^n] = nx^{n-1}$. You may use the product rule: $\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$.

For problems 8 - 9, prove one using Mathematical Induction:

8. $(1 + x)^n \geq 1 + nx$ for any natural number n and any real number $x \geq -1$.

9. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that:
 $|x_1 + x_2 + x_3 + \dots + x_n| \leq |x_1| + |x_2| + |x_3| + \dots + |x_n|$ (the General Triangle Inequality).