MTH 6610 - Final Exam - Answers

TERM V - 2022

Pat Rossi Name ______________________

1. Just as learning, and particularly mathematics, was being revived in the 13 Century, what event(s) brought an immediate end to the learning in the 14th Century?

   In short: famine, plague, war, and death. Significant climate change resulted in constant, heavy rainfalls over an extended period of time, which coincided with the onset of a pronounced, centuries-long drop in the average yearly temperature, known as the "Little Ice Age." The resulting crop failures and famine led to a mortality rate (in many places) of about 10%. The survivors of the famine, weakened by malnutrition, lacked resistance to disease. In particular, the Black Death (Bubonic Plague) spread throughout much of Western Europe from 1347-48, and reached England in 1349. It is estimated that 30-60% of Europe's total population perished. Even after the plague subsided, it re-emerged at intervals of 12-15 years until the late seventeenth century.

   The 14th Century was also a period of constant warfare and uprisings in Europe. The most notable of these "wars" was the Hundred Years War (Between England and France), which lasted from 1338-1452.

2. What event and two developments (inventions) are credited with reviving learning (and mathematics) in the 15th Century?

   The fall of Constantinople to the Turks in 1453; John Gutenberg's invention of the printing press, with movable, metallic type (circa 1450); and the ability to produce paper on a large scale. The collapse of the Byzantine Empire resulted in a mass exodus of Greek scholars who sought refuge in Italy. These scholars brought with them a treasure of many ancient manuscripts, either unknown to the West, or known only via Arabic translations.

   The invention of the printing press and the ability to produce paper on a large scale made (newly) acquired knowledge and information accessible to a large audience.

3. What was Francois Vieta's innovation, and how did it help to further the study of algebra?

   Vieta's innovation of using consonants to represent constants (known quantities) and vowels to represent unknown quantities made it possible to solve an *entire class* of equations at once. (e.g., the solution of the "generic" quadratic equation $ax^2+bx+c=0$), in essence provided the solution for *all* quadratic equations.)

4. What trick did Cardano use to solve general cubic equations?

   He used a substitution trick to convert the generic cubic equation $x^3+ax^2+bx+c=0$ into a cubic equation in which the second degree term was absent. The substitution:

$$x = y - \frac{a}{3}$$

   transformed the generic cubic equation $x^3 + ax^2 + bx + c = 0$ into the cubic equation:

$$y^3 + \left(b - \frac{a^2}{3}\right) y + \left(\frac{2a^3}{27} - \frac{ab}{3} + c\right) = 0.$$

5. What mathematical fact was established by Paolo Ruffini and Niels Able? What does this really mean?

   It is impossible to find an algebraic solution to the general fifth degree equation:

$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0.$$

   It is impossible to derive an equation, a set of equations, or an algorithm that will yield the solutions of all fifth degree equations.

6. For what contribution to mathematics is John Napier credited?

   The invention of logarithms.

7. What mathematical pioneers are generally credited with "paving the way" for the development of Calculus?

   Cavalieri, Torricelli, Barrow, Descartes, Fermat, and Wallis.

8. How did Newton acknowledge the previous fact?

   He made the statement: "If I have seen farther than others, it is because I have stood on the shoulders of giants."

9. What do the words "geocentric" and "heliocentric" mean, and where do the words come from?

   "geocentric" means "earth centered" and "heliocentric" means "sun centered."

   It had been the viewpoint of theologians and ancient astronomers and philosophers alike, that earth was the center of the universe, and that the sun revolved around the earth. (Hence, the universe was thought to be "geocentric.")

   Nicolaus Copernicus had observed among other things that the earth (and other planets) revolve about the sun (not vice versa). Astronomer Galileo confirmed this as fact. (Hence, the solar system is, in fact, "heliocentric.")

10. What explanation is given for the drastic increase in mathematical Achievements during the 16th and 17th Centuries?

    The introduction of the symbols $+$, $-$, $=$, $\cdot$, $\times$, $\div$, $\sqrt{}$, $<$, and $>$, to name a few, facilitated algebraic manipulation. For example, image how hard it would be to solve a simple problem if there were no such thing as an equal sign !!!

11. What advance was made by Simon Stevin?

    The introduction of decimal notation (i.e., "decimal fractions")

12. What improvement of an already existing mathematical tool was made by Henry Briggs?

    The existing tool was the logarithm. The improvement was that of establishing the logarithm of 1 to be zero and the logarithm of 10 to be 1 (i.e., Briggs developed the logarithm base 10). (Recall that $\log_{10}(1) = 0$ and $\log_{10}(10) = 1$.)

13. What do Jobst Burgi and John Napier have in common?

    Burgi, independent of Napier, conceived the idea of logarithms (circa 1580).

14. What was an "immediate" technological consequence of Napier's work?

    William Oughtred invented the slide rule in 1622. The slide rule was a computational device (with one (later two) moving parts) that made copious use of logarithmic scales (as opposed to linear scales).

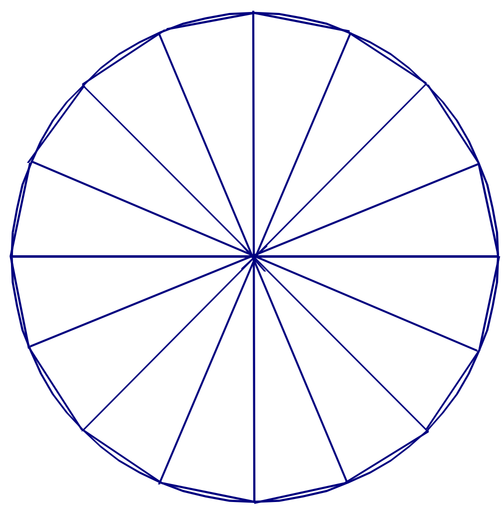
15. How did Kepler approach the computation of the area of a circle and the volume of a sphere (Be somewhat specific here).

Kepler found the area of a circle by imagining the circle to be made up of infinitely many triangles having their common vertex at the center of the circle and their infinitesimally small bases lying on the perimeter of the circle (as shown below). Actually, he started out with n identical triangles. Since the area of such a triangle can be computed, the area A of the circle would be approximately equal to the sum of the areas of the triangles,

$$A \approx \sum_{i=1}^{n} \left(\text{area of i}^{\text{th}} \text{ triangle}\right)$$

the area of the circle would turn out to be the limit of the sum as $n \to \infty$.

$$\text{i.e., } A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\text{area of i}^{\text{th}} \text{ triangle}\right)$$



Analogously, Kepler envisioned a sphere to be made up of infinitely many identical 5-sided pyramids (four congruent, triangular sides, plus a square base) with their common vertex at the center of the sphere, and their bases on the inner surface of the sphere.

16. What laws of planetary motion were advanced by Kepler, and what does the second law mean?

    1. The planets move in elliptical orbits with the sun at one focus.

    2. Each planet moves along its orbit in such a way that a line drawn from the sun to the planet sweeps out equal areas in equal time intervals.

    3. The square of the time required for any planet to make a complete orbit around the sun is proportional to the cube of its mean distance from the sun.

    The second law suggests that the orbital velocity of a planet decreases to its minimum as the planet approaches the sun and increases to its maximum velocity as the planet moves away from the sun.

17. For what Philosophical "maxim" is Rene Descartes known, and (briefly) how do you explain the meaning?

    "I think, therefore I am." The fact that a thought exists necessitates the existence of the person who thought the thought.

18. What was Descartes' most important contribution to mathematics?

    The science of Analytic Geometry, for which he laid the groundwork by inventing the Cartesian Plane (aka the Cartesian Coordinate System, aka the x-y plane), as well as the 3-dimensional analog (with x, y, and z axes).

19. Descartes work inspired the use of what mathematical tool, common to all who study algebra today?

    The invention of the Cartesian Plane (aka the Cartesian Coordinate System, aka the x-y plane), as well as the 3-dimensional analog (with x, y, and z axes).

20. What two observations, regarding the roots of polynomials, did Descartes make?

    I'll give three observations:

    A polynomial $P(x)$ has $(x-a)$ as a factor if and only if a is a root of $P(x)$ (i.e., if and only if $P(a)=0$).

    Any polynomial equation of degree n has n roots.

    The number of positive roots of the equation

    $$f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0$$

    is equal either to the number of variations of signs in its coefficients or to this number decreased by a positive integer.

21. What association more than likely influenced Isaac Newton's development of Calculus?

    Newton's mentor and "department chair," Isaac Barrow, did a great deal of work and research in the areas of drawing tangents to curves and computing areas under curves. This exposed Newton to mathematical considerations that certainly made it likely that he would end up working on pursuits that could very naturally lead to the development of calculus.

22. What three "discoveries" did Isaac Newton make, while on a two year exile from London, for the purpose of avoiding "the plague"?

    1. The development of the "Method of Fluxions," which today is known as differential calculus.

    2. The analysis of white light as being the composite sum of light of all "colors of the spectrum,"

    3. The concept and formulation of the law of universal gravitation. (The gravitational force between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them.)

23. What is the basic idea behind Newton's law regarding planetary motion?

    The path of a body (e.g., planet) defines an ellipse, under the influence of an attractive force at one focus, if and only if the force varies inversely with the square of the distance between the object and the focus.

24. What later "theoretical breakthrough," regarding gravity, allowed Newton to put the finishing touches on his law, regarding planetary motion? (This is something that Newton had ASSUMED to be true, earlier on, but was unable to prove at that time.)

    The gravitational pull of a spherical body is the same as if the sphere's entire mass were located (as a "point mass") at the sphere's center.

25. What reasons can be given for the fact that Newton's development of calculus was not acknowledged right away?

    Newton developed calculus in his quest to explain the "celestial mechanics" of planetary motion, universal gravitation, etc., felt it best to explain his results in terms of classical Greek geometry - something that was understood by his "target audience,' rather than use fluxions, which was not known by any of his colleagues, much less understood by a majority of them.

26. How are Isaac Newton and Gottfried Leibniz tied together historically?

    Both developed the discipline of Calculus simultaneously.

27. To what extent did Leibniz make progress in the development of his contribution to mathematics? (i.e., What results, well known today, did he derive?)

    The Fundamental Theorem of Calculus (relating Differential and Integral Calculus), the power rule for derivatives, the product rule, the quotient rule, and integration by parts.

28. What contribution, made by Jean d'Alembert, relates to the common field of study of Newton and Leibniz?

    The theory of limits is fundamental to calculus.

29. What "international feud" resulted from the work of Newton and Leibniz, and how did this effect mathematicians on both sides? What was the result?

    The feud centered around which mathematician – Newton or Liebniz – had the right to lay claim to inventing calculus. English mathematicians supported Newton's claim while the continental mathematicians of Europe supported Liebniz' claim. This feud turned so bitter, that for more than a century, there was virtually no collaboration and little communication between British mathematicians and continental European mathematicians. Both sides suffered from this self-imposed isolationism, but the British suffered especially.