MTH 3318 - Test #2

FALL 2014

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Instructions.   Fully document your work.

1. In exercises 1.a - 1.d, let p be the statement: "We have plenty of rain," and let q be the statement: "our flowers will grow." Write each statement in symbolic form.

   (a) If we have plenty of rain, then our flowers will grow.

   (b) We will have plenty of rain, or our flowers will not grow.

   (c) Our having plenty of rain is a necessary and sufficient condition for our flowers to grow.

   (d) We will have plenty of rain if our flowers grow.

2. In exercises 2.a - 2.d, let p be the statement: "$f(x)$ is continuous" and let q be the statement: "$f(x)$ is differentiable." Write each statement in words.

   (a) $p \wedge q$

   (b) $p \vee q$

   (c) $q \rightarrow \sim p$

   (d) $\sim p \leftrightarrow \sim q$

3. In problems 3.a - 3.d, determine whether the given propositions are True or False:

(a) If $3^2 = 9$, then $8 > 10$.

(b) If $3^2 > 3$, then $3^2 > 5$.

(c) If $3^2 > 10$ if and only if $3^2 = 5$.

(d) If $3^2 = 5$, then $3^2 > 10$.

4. In exercises 4.a-4.b construct a truth table for the statement given.

(a) $p \vee (q \longleftrightarrow r)$

(b) $(\sim p \longrightarrow q) \wedge \sim r$

5. For problems 5.a - 5.d, negate the given statements:

   (a) All toads have warts.

   (b) Some birds can fly.

   (c) Some submarines do not have screen doors.

   (d) $\forall$ real numbers x, $\exists$ real number y, $\ni x + y = x$.
       (i.e. For all real numbers x, there exists a real number y such that $x + y = x$.)

6. For problems 6.a - 6.b, disprove the given statements by providing a suitable counter-example:

   (a) $\forall n \in \mathbb{N}$, if $2n + 1$ is prime, then n is even.

   (b) For all integers x, y, and z, if x is a factor of $(y + z)$, then x is a factor of y and x is a factor of z.

7. Write the converse, inverse, and contrapositive of the following statement, labeling each one.

   If I eat my vegetables, then I will grow "big and strong."

8. In problems 8.a - 8.b, determine whether the given arguments are valid.

   (a) I will go to the concert if and only if I finish studying. If I get to bed late, then I will have finished studying. Therefore, I will have been to the concert if I go to bed late.

   (b) Some desks are made of wood. All paper is made of wood. Therefore, some desks are made of paper.

9. In problems 9.a - 9.b, determine whether the given arguments are valid.

   (a) If I'm thrifty and I save my money, then I will buy a bicycle. I will buy a bicycle. Therefore, if I don't buy a bicycle, then I will not have been thrifty.

   (b) No ducks are birds. Some animals are birds. Therefore, no ducks are animals.