MTH 1125 Test #1 - Solutions

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Pat Rossi Name ______________________________

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to\ 2} \frac{2x^2-3x+4}{x^2+3} =$

Step #1 Try Plugging In:

$\lim_{x\to\ 2} \frac{2x^2-3x+4}{x^2+3} = \frac{2(2)^2-3(2)+4}{(2)^2+3} = \frac{6}{7}$

i.e., $\lim_{x\to\ 2} \frac{2x^2-3x+4}{x^2+3} = \frac{6}{7}$

2. Compute: $\lim_{x\to\ 2} \frac{x^2-5x+6}{x^2-7x+10} =$

$\lim_{x\to\ 2} \frac{x^2-5x+6}{x^2-7x+10} = \frac{(2)^2-5(2)+6}{(2)^2-7(2)+10} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Canceling:

$\lim_{x\to\ 2} \frac{x^2-5x+6}{x^2-7x+10} = \lim_{x\to\ 2} \frac{(x-3)(x-2)}{(x-5)(x-2)} = \lim_{x\to\ 2} \frac{(x-3)}{(x-5)} = \frac{(2)-3}{(2)-5} = \frac{-1}{-3} = \frac{1}{3}$

i.e., $\lim_{x\to\ 2} \frac{x^2-5x+6}{x^2-7x+10} = \frac{1}{3}$

3. $\lim_{x\to\ -2} \frac{x^2+4x-9}{x^2-x-6}$

Step #1 Try Plugging in:

$$\lim_{x\to\ -2} \frac{x^2+4x-9}{x^2-x-6} = \frac{(-2)^2+4(-2)-9}{(-2)^2-(-2)-6} = \frac{-13}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Canceling:

No Good! "Factoring and Canceling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ -2^-} \frac{x^2+4x-9}{x^2-x-6} = \lim_{x\to\ -2^-} \frac{x^2+4x-9}{(x-3)(x+2)} = \frac{-13}{(-5)(-\varepsilon)} = \frac{\left(\frac{-13}{-5}\right)}{(-\varepsilon)} = \frac{\left(\frac{13}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < - \\ \Rightarrow & x+2 < 0 \end{array}$$

$$\lim_{x\to\ -2^+} \frac{x^2+4x-9}{x^2-x-6} = \lim_{x\to\ -2^+} \frac{x^2+4x-9}{(x-3)(x+2)} = \frac{-13}{(-5)(+\varepsilon)} = \frac{\left(\frac{-13}{-5}\right)}{(+\varepsilon)} = \frac{\left(\frac{13}{5}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ -2} \frac{x^2+4x-9}{x^2-x-6}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ \infty} \frac{3x^5-3x^3+3}{2x^4+6x+4} =$

$$\lim_{x\to\ \infty} \frac{3x^5-3x^3+3}{2x^4+6x+4} = \lim_{x\to\ \infty} \frac{3x^5}{2x^4} = \lim_{x\to\ \infty} \frac{3x}{2} = \lim_{x\to\ \infty} \frac{3}{2}x = \infty$$

i.e., $\lim_{x\to\ \infty} \frac{3x^5-3x^3+3}{2x^4+6x+4} = \infty$

5. Find the asymptotes and graph: $f(x) = \frac{x^2+5x+4}{x^2-9}$

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 9 = 0$

$\Rightarrow (x+3)(x-3) = 0$

$\Rightarrow x = -3$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -3^-} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to\ -3^-} \frac{x^2+5x+4}{(x+3)(x-3)} = \frac{-2}{(-\varepsilon)(-6)} = \frac{2}{(-\varepsilon)(6)} = \frac{\left(\frac{2}{6}\right)}{-\varepsilon} = -\infty$$

$$\begin{array}{|ll|} & x \to -3^- \\ \Rightarrow & x < -3 \\ \Rightarrow & x+3 < 0 \\ \end{array}$$

$$\lim_{x\to\ -3^+} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to\ -3^+} \frac{x^2+5x+4}{(x+3)(x-3)} = \frac{-2}{(+\varepsilon)(-6)} = \frac{2}{(+\varepsilon)(6)} = \frac{\left(\frac{2}{6}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|} & x \to -3^+ \\ \Rightarrow & x > -3 \\ \Rightarrow & x+3 > 0 \\ \end{array}$$

Since the one-sided limits are infinite, $x = -3$ *is a vertical asymptote.*

$$\lim_{x\to\ 3^-} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to\ 3^-} \frac{x^2+5x+4}{(x+3)(x-3)} = \frac{28}{(6)(-\varepsilon)} = \frac{\left(\frac{28}{6}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{|ll|} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \\ \end{array}$$

$$\lim_{x\to 3^+} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to 3^+} \frac{x^2+5x+4}{(x+3)(x-3)} = \frac{28}{(6)(+\varepsilon)} = \frac{\left(\frac{28}{6}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{|ll|} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \\ \end{array}$$

Since the one-sided limits are **infinite,** $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to\ -\infty} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to\ -\infty} \frac{x^2}{x^2} = \lim_{x\to\ -\infty} 1 = 1$

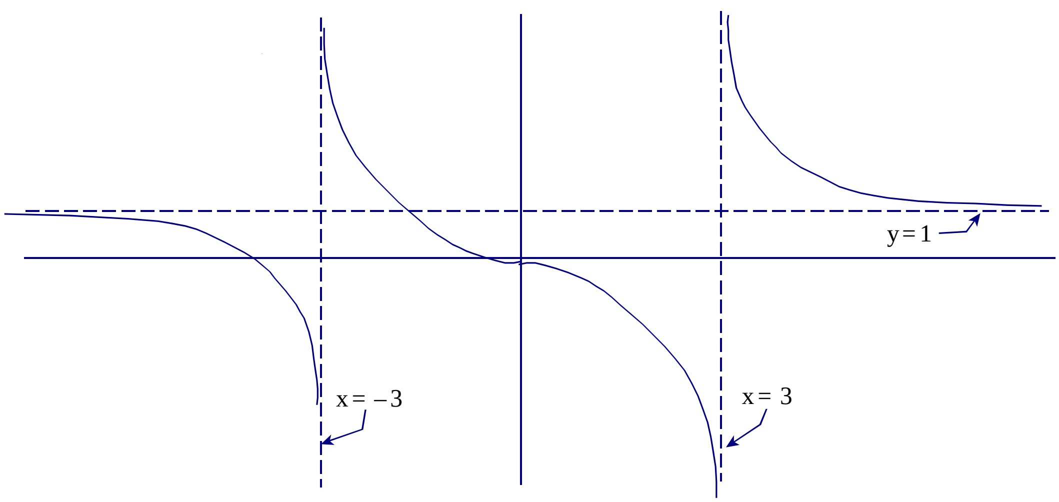
$\lim_{x\to+\infty} \frac{x^2+5x+4}{x^2-9} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-3^-} \frac{x^2+5x+4}{x^2-9} = -\infty$	
$\lim_{x\to-3^+} \frac{x^2+5x+4}{x^2-9} = +\infty$	$\lim_{x\to\ -\infty} \frac{x^2+5x+4}{x^2-9} = 1$
$\lim_{x\to3^-} \frac{x^2+5x+4}{x^2-9} = -\infty$	$\lim_{x\to+\infty} \frac{x^2+5x+4}{x^2-9} = 1$
$\lim_{x\to3^+} \frac{x^2+5x+4}{x^2-9} = +\infty$	

Graph $f(x) = \frac{x^2-5x+4}{x^2-9}$



6. Compute: $\lim_{x\to5}\frac{\sqrt{x-1}-2}{x-5}=$

Step #1 Try Plugging in:

$$\lim_{x\to5}\frac{\sqrt{x-1}-2}{x-5}=\frac{\sqrt{(5)-1}-2}{(5)-5}=\frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to5}\frac{\sqrt{x-1}-2}{x-5}=\lim_{x\to5}\frac{\sqrt{x-1}-2}{x-5}\cdot\frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}=\lim_{x\to5}\frac{\left(\sqrt{x-1}\right)^2-(2)^2}{(x-5)\left[\sqrt{x-1}+2\right]}$$

$$=\lim_{x\to5}\frac{(x-1)-4}{(x-5)\left[\sqrt{x-1}+2\right]}=\lim_{x\to5}\frac{(x-5)}{(x-5)\left[\sqrt{x-1}+2\right]}=\lim_{x\to5}\frac{1}{\left[\sqrt{x-1}+2\right]}$$

$$=\frac{1}{\left[\sqrt{5-1}+2\right]}=\frac{1}{[2+2]}=\frac{1}{4}$$

i.e., $\lim_{x\to5}\frac{\sqrt{x-1}-2}{x-5}=\frac{1}{4}$

7.

$x=$	$f(x)=$
2.5	10.2
2.9	157.32
2.99	10045.56
2.999	$235,402.27$
2.9999	$5,873,002.16$

$x=$	$f(x)=$
3.5	10.2
3.1	157.32
3.01	10045.56
3.001	$235,402.27$
3.0001	$5,873,002.16$

Based on the information in the table above, do the following:

(a) $\lim_{x\to3^-}f(x)=+\infty$

(b) $\lim_{x\to3^+}f(x)=+\infty$

(c) Graph $f(x)$