MTH 1125 - Test 2 (12pm Class) - Solutions

Fall 2016

Pat Rossi Name ______________________

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[4x^6+6x^4+8x^3+24x+2\sqrt{x}+6\right]=$

$\frac{d}{dx}\left[4x^6+6x^4+8x^3+24x+2x^{\frac{1}{2}}+6\right]$

$=4\left[6x^5\right]+6\left[4x^3\right]+8\left[3x^2\right]+24+2\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

$=24x^5+24x^3+24x^2+24+x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[4x^6+6x^4+8x^3+24x+2\sqrt{x}+6\right]=24x^5+24x^3+24x^2+24+x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[\cos(x)\cot(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{\cos(x)}_{1^{st}}\underbrace{\cot(x)}_{2^{nd}}\right]=\underbrace{(-\sin(x))}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\cot(x)}_{2^{nd}}+\underbrace{\left(-\csc^2(x)\right)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\cos(x)}_{1^{st}}$$

$\frac{d}{dx}\left[\cos(x)\cot(x)\right]=-\sin(x)\cot(x)-\csc^2(x)\cos(x)$

3. Compute: $\frac{d}{dx}\left[\frac{\sec(x)}{6x^2-12x+5}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{\sec(x)}^{\text{top}}}{\underbrace{6x^2-12x+5}_{\text{Bottom}}}\right]=\frac{\overbrace{(\sec(x)\tan(x))}^{\text{top prime}}\cdot\overbrace{\left(6x^2-12x+5\right)}^{\text{bottom}}-\overbrace{(12x-12)}^{\text{bottom prime}}\cdot\overbrace{\sec(x)}^{\text{top}}}{\underbrace{\left(6x^2-12x+5\right)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{\sec(x)}{6x^2-12x+5}\right]=\frac{\sec(x)\tan(x)\left(6x^2-12x+5\right)-(12x-12)\sec(x)}{\left(6x^2-12x+5\right)^2}$

4. Compute: $\frac{d}{dx}\left[\left(4x^6+2x^3\right)^{15}\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(4x^6+2x^3\right)^{15}\right] = \underbrace{15\left(4x^6+2x^3\right)^{14}}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(24x^5+6x^2\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(4x^6+2x^3\right)^{15}\right] = 15\left(4x^6+2x^3\right)^{14}\left(24x^5+6x^2\right)$

5. Given that $f(x) = x^2+2x+1$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(1,4)$.

   We need two things:

   i. A point on the line (We have that: $(x_1, y_1) = (1,4)$)

   ii. The slope of the line (This is $f'(x_1)$)

   $f'(x) = 2x+2$

   At the point $(x_1, y_1) = (1,4)$, **the slope is** $f'(1) = 2(1)+2 = 4$

   We will use the Point-Slope equation of a line:

   $y - y_1 = m(x - x_1)$ (Where m is the slope and (x_1, y_1) is a known point on the line.)

   Thus, the equation of the line tangent to the graph of $f(x)$ is:

   $y - 4 = 4(x-1)$

The equation of the line tangent is $y - 4 = 4(x-1)$

6. Given that $y = 5x^3 + 5x^2$ and that $x = \tan(t)$; compute $\frac{dy}{dt}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

We know:

$\frac{dy}{dx} = 15x^2 + 10x$

$\frac{dx}{dt} = \sec^2(t)$

We want: $\frac{dy}{dt}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (15x^2 + 10x)\sec^2(t) = \underbrace{\left(15(\tan(t))^2 + 10(\tan(t))\right)\sec^2(t)}_{\substack{\text{express solely in terms of} \\ \text{independent variable } t}}$$

i.e. $\frac{dy}{dt} = (15\tan^2(t) + 10\tan(t))\sec^2(t)$

7. Compute: $\frac{d}{dx}[\cot(x^2 + 2x + 3)] =$

Outer:	=	cot ( )
Deriv. of outer	=	$-\csc^2$ ( )

$$\frac{d}{dx}\left[\begin{array}{c} \cot\left(\underbrace{x^2 + 2x + 3}\right) \\ \uparrow \qquad\qquad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{-\csc^2\left(x^2 + 2x + 3\right)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{(2x + 2)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

i.e., $\frac{d}{dx}[\cot(x^2 + 2x + 3)] = -\csc^2(x^2 + 2x + 3)(2x + 2)$

8. Compute: $\frac{d}{dx}\left[\left(\frac{3x^2+9x}{8x^5+8x}\right)^4\right] =$

$$\frac{d}{dx}\left[\underbrace{\left(\frac{3x^2+9x}{8x^5+8x}\right)^4}_{(g(x))^n}\right] = \underbrace{4\left(\frac{3x^2+9x}{8x^5+8x}\right)^3}_{\substack{\text{power rule}\\\text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{3x^2+9x}{8x^5+8x}\right]\right)}_{\substack{\text{deriv of}\\\text{inner Function}}}$$

$$= 4\left(\tfrac{3x^2+9x}{8x^5+8x}\right)^3 \underbrace{\frac{(6x+9)(8x^5+8x) - (40x^4+8)(3x^2+9x)}{(8x^5+8x)^2}}_{\substack{\text{quotient}\\\text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{3x^2+9x}{8x^5+8x}\right)^4\right] = 4\left(\frac{3x^2+9x}{8x^5+8x}\right)^3 \cdot \frac{(6x+9)\left(8x^5+8x\right)-\left(40x^4+8\right)\left(3x^2+9x\right)}{(8x^5+8x)^2}$

9. Compute: $\frac{d}{dx}\left[\sin^{10}(5x^3+15x)\right] =$ Re-write!

$\frac{d}{dx}\left[(\sin(5x^3+15x))^{10}\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[(\sin(5x^3+15x))^{10}\right] = \underbrace{10\left(\sin\left(5x^3+15x\right)\right)^9}_{\substack{\text{power rule}\\\text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\sin\left(5x^3+15x\right)\right]\right)}_{\substack{\text{derivative}\\\text{of inner}}}$$

$$= 10(\sin(5x^3+15x))^9 \cdot \underbrace{\left(\cos\left(5x^3+15x\right)\right) \cdot \left(15x^2+15\right)}_{\substack{\text{Chain}\\\text{Rule}}}$$

i.e., $\frac{d}{dx}\left[\sin^{10}(5x^3+15x)\right] = 10(\sin(5x^3+15x))^9(\cos(5x^3+15x)) \cdot (15x^2+15)$

10. Given that $L'(x) = \frac{1}{x}$; compute $\frac{d}{dx}[L(\tan(x))]$

Outer:	$=$	$L(\quad)$
Deriv. of outer	$=$	$\frac{1}{(\quad)}$

$$\frac{d}{dx}\left[\underset{\substack{\uparrow \\ \text{outer}}}{L}\left(\underset{\substack{\uparrow \\ \text{inner}}}{\underbrace{\tan(x)}}\right)\right] = \underbrace{\frac{1}{(\tan(x))}}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\sec^2(x)}_{\substack{\text{deriv. of} \\ \text{inner}}} = \frac{\sec^2(x)}{\tan(x)}$$

i.e., $\frac{d}{dx}[L(\tan(x))] = \frac{1}{\tan(x)} \cdot \sec^2(x) = \frac{\sec^2(x)}{\tan(x)}$

11. Given that $f(x) = 4x^2 - x + 3$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[4(x+\Delta x)^2 - (x+\Delta x) + 3\right] - \left[4x^2 - x + 3\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[4\left(x^2 + 2x\Delta x + \Delta x^2\right) - (x+\Delta x) + 3\right] - \left[4x^2 - x + 3\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[4x^2 + 8x\Delta x + 4\Delta x^2 - x - \Delta x + 3\right] - \left[4x^2 - x + 3\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{8x\Delta x + 4\Delta x^2 - \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(8x + 4\Delta x - 1)}{\Delta x} = \lim_{\Delta x \to 0} (8x + 4\Delta x - 1) = 8x + 4(0) - 1 = 8x - 1$$

i.e., $f'(x) = 8x - 1$

12. Given that $x^4 + 5y^3 = 3x^4y^3$; compute y'

i. Differentiate both sides w.r.t. x

$$\frac{d}{dx}\left[x^4 + 5y^3\right] = \frac{d}{dx}\left[\underbrace{3x^4}_{1^{\text{st}}}\underbrace{y^3}_{2^{\text{nd}}}\right]$$

$$\Rightarrow 4x^3 + 15y^2 \cdot y' = \underbrace{12x^3}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^3}_{2^{\text{nd}}} + \underbrace{3y^2 \cdot y'}_{2^{\text{nd}}\text{prime}} \cdot \underbrace{3x^4}_{1^{\text{st}}}$$

ii. Solve algebraically for y'

a. Get y' terms on left side, all other terms on right side

$$\Rightarrow 15y^2y' - 3y^2y' \cdot 3x^4 = 12x^3y^3 - 4x^3$$

b. Factor out y'

$$\Rightarrow \left(15y^2 - 3y^23x^4\right)y' = 12x^3y^3 - 4x^3$$

c. Divide both sides by the cofactor of y'

$$y' = \frac{12x^3y^3-4x^3}{15y^2-3y^23x^4} = \frac{12x^3y^3-4x^3}{15y^2-9y^2x^4}$$

$$\boxed{y' = \frac{12x^3y^3-4x^3}{15y^2-3y^23x^4} = \frac{12x^3y^3-4x^3}{15y^2-9y^2x^4}}$$