

MTH 4441 Exercises To Study for Test #1

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Pat Rossi

Name _____

1. In each case below, determine whether $*$ is a **closed** binary operation on the given set. If it *IS* a **closed** binary operation, then determine whether it is commutative and/or associative.

(a) $(\mathbb{Z}, *)$ where $a * b = a + b^2$

(b) $(\mathbb{Z}, *)$ where $a * b = a^2b^3$

(c) $(\mathbb{R}, *)$ where $a * b = \frac{a}{a^2+b^2}$

(d) $(\mathbb{Z}, *)$ where $a * b = \frac{a^2+2ab+b^2}{a+b}$

(e) $(\mathbb{Z}, *)$ where $a * b = a + b - ab$

(f) $(\mathbb{R}, *)$ where $a * b = b$

(g) $(\{-4, -2, 1, 2, 3\}, *)$ where $a * b = |b|$

(h) $(\{1, 2, 3, 6, 18\}, *)$ where $a * b = ab$

(i) $\left(\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}, * \right)$ where $*$ is matrix addition

2. Let $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, and let (\mathbb{Z}_6, \oplus) be a group, where \oplus is addition modulo 6. Construct the group table.

3. In the group (\mathbb{Z}_6, \oplus) , what is the order of the element 2? What is the order of the element 3?

(i.e., $o(2) = ?$ $o(3) = ?$)

4. Construct the group table for (\mathbb{Z}_7, \oplus) .
5. Let $U_5 = \{1, 2, 3, 4\}$, and let (U_5, \odot) be a group, where \odot is multiplication modulo 5. Construct the group table.
6. Construct the group table for (U_3, \odot) .
7. Construct the group table for (U_7, \odot) .
8. Construct the group table for (U_6, \odot) .

(a) (U_6, \odot) is NOT a group. Give at least two reasons why it is not a group

9. Construct the group table for (U_4, \odot) .

(a) (U_4, \odot) is NOT a group. Give at least two reasons why it is not a group

10. Determine whether the table below defines a group for $G = \{a, b, c\}$. (State why or why not.)

*	a	b	c
a	a	b	c
b	b	a	c
c	c	b	a

11. Determine whether the table below defines a group for $G = \{a, b, c\}$. (State why or why not.)

*	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

12. Determine whether the table below defines a group for $G = \{a, b, c, d, e, f\}$. State why or why not. (You may assume that the operation $*$ is associative.)

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	d	f	a	c	e
c	c	f	b	e	a	d
d	d	a	e	b	f	c
e	e	c	a	f	d	b
f	f	e	d	c	b	a

13. In the previous exercise, what is the inverse of d ? How do you know?
14. Compute the remainder of 25 modulo 7 (i.e. $25 \equiv \underline{\hspace{1cm}} \pmod{7}$)
15. Compute the remainder of 48 modulo 5 (i.e. $48 \equiv \underline{\hspace{1cm}} \pmod{5}$)
16. Compute the remainder of 53 modulo 14 (i.e. $53 \equiv \underline{\hspace{1cm}} \pmod{14}$)
17. Determine whether 58 and 75 are congruent modulo 9 (Determine whether $58 \equiv 75 \pmod{9}$)
18. Determine whether 43 and 59 are congruent modulo 16 (Determine whether $43 \equiv 59 \pmod{9}$)
19. Compute $\gcd(4, 18)$
20. Compute $\gcd(25, 40)$
21. Compute $\gcd(4, 25)$