MTH 4441 Test #3

FALL 2021

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1. **Define - permutation**

2. **Define -** r**-cycle** (or **cycle**).

3. **Prove:** Let $S = \{1, 2, 3, \ldots, n\}$ and let S_n be the set of all permutations $f : S \to S$. Furthermore, let $\circ$ be the operation of function composition. Then $(S_n, \circ)$ is a group.

4. **Define - disjoint cycles**

5. **Define - transposition**

6. For Exercises 6-7, State two theorems about permutations.

7.

8. Perform the indicated operations in S_6

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 6 & 4 & 5 \end{pmatrix} =$$

9. Express the permutation as a "product" of disjoint cycles and then as the "product" of transpositions. Classify the permutation as being either **even** or **odd.**

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 2 & 1 & 6 & 5 & 8 & 7 \end{pmatrix} =$$

10. Given $(U_5, \odot) = (\{1, 2, 3, 4\}, \odot)$, construct a group of permutations on U_5 that is isomorphic to $(U_5, \odot)$, and exhibit an isomorphism from $(U_5, \odot)$ to this group.

11. Consider the group $(G,*)$ given in the table below:

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Construct a group of permutations on G that is isomorphic to $(G,*)$, and exhibit an isomorphism from $(G,*)$ to this group.

12. We are given a group $(G,*)$, and an element $x \in G$. Given also that $x^5 = e$ and that $x^3 = e$, prove that $x = e$.