MTH 4441 Exercises To study for Test #1

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1. In each case below, determine whether $*$ is a **closed** binary operation on the given set. If it *IS* a **closed** binary operation, then determine whether it is commutative and/or associative.

   (a) $(\mathbb{Z}, *)$ where $a * b = a + b^2$

   (b) $(\mathbb{Z}, *)$ where $a * b = a^2 b^3$

   (c) $(\mathbb{R}, *)$ where $a * b = \frac{a}{a^2+b^2}$

   (d) $(\mathbb{Z}, *)$ where $a * b = \frac{a^2+2ab+b^2}{a+b}$

   (e) $(\mathbb{Z}, *)$ where $a * b = a + b - ab$

   (f) $(\mathbb{R}, *)$ where $a * b = b$

   (g) $(\{-4, -2, 1, 2, 3\}, *)$ where $a * b = |b|$

   (h) $(\{1, 2, 3, 6, 18\}, *)$ where $a * b = ab$

   (i) $\left(\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}\right\}, *\right)$ where $*$ is matrix addition

2. Let $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, and let $(\mathbb{Z}_6, \oplus)$ be a group, where $\oplus$ is addition modulo 6. Construct the group table.

3. In the group $(\mathbb{Z}_6, \oplus)$, what is the order of the element 2? What is the order of the element 3?

   (i.e., $o(2) = ?$ $\quad o(3) = ?$)

4. Construct the group table for $(\mathbb{Z}_7, \oplus)$.

5. Let $U_5 = \{1, 2, 3, 4\}$, and let $(U_5, \odot)$ be a group, where $\odot$ is multiplication modulo 5. Construct the group table.

6. Construct the group table for $(U_3, \odot)$.

7. Construct the group table for $(U_7, \odot)$.

8. Construct the group table for $(U_6, \odot)$.

   (a) $(U_6, \odot)$ is NOT a group. Give at least two reasons why it is not a group

9. Construct the group table for $(U_4, \odot)$.

   (a) $(U_4, \odot)$ is NOT a group. Give at least two reasons why it is not a group

10. Determine whether the table below defines a group for $G = \{a, b, c\}$. (State why or why not.)

*	a	b	c
a	a	b	c
b	b	a	c
c	c	b	a

11. Determine whether the table below defines a group for $G = \{a, b, c\}$. (State why or why not.)

*	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

12. Determine whether the table below defines a group for $G = \{a, b, c, d, e, f\}$. State why or why not. (You may assume that the operation $*$ is associative.)

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	d	f	a	c	e
c	c	f	b	e	a	d
d	d	a	e	b	f	c
e	e	c	a	f	d	b
f	f	e	d	c	b	a

13. In the previous exercise, what is the inverse of d? How do you know?

14. Compute the remainder of 25 modulo 7 (i.e. $25 \equiv \_\_\_ \pmod{7}$)

15. Compute the remainder of 48 modulo 5 (i.e. $48 \equiv \_\_\_ \pmod{5}$)

16. Compute the remainder of 53 modulo 14 (i.e. $53 \equiv \_\_\_ \pmod{14}$)

17. Determine whether 58 and 75 are congruent modulo 9 (Determine whether $58 \equiv 75 \pmod{9}$)

18. Determine whether 43 and 59 are congruent modulo 16 (Determine whether $43 \equiv 59 \pmod{9}$)

19. Compute $\gcd(4, 18)$

20. Compute $\gcd(25, 40)$

21. Compute $\gcd(4, 25)$