# MTH 1125 Test #1

FALL 2009

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^3-3x-3}{x^2+x+2}=$

   (a) 1. Try Plugging in:

   $\lim_{x\to 2}\frac{x^3-3x-3}{x^2+x+2}=\frac{(2)^3-3(2)-3}{(2)^2+(2)+2}=-\frac{1}{8}$

   i.e., $\lim_{x\to 2}\frac{x^3-3x-3}{x^2+x+2}=-\frac{1}{8}$

2. Compute: $\lim_{x\to 3}\frac{x^2-6x+9}{x^2+x-12}=$

   (a) 1. Try Plugging in:

   $\lim_{x\to 3}\frac{x^2-6x+9}{x^2+x-12}=\frac{(3)^2-6(3)+9}{(3)^2+(3)-12}=\frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

   $\lim_{x\to 3}\frac{x^2-6x+9}{x^2+x-12}=\lim_{x\to 3}\frac{(x-3)^2}{(x+4)(x-3)}=\lim_{x\to 3}\frac{(x-3)}{(x+4)}=\frac{(3)-3}{(3)+4}=\frac{0}{7}=0$

   i.e., $\lim_{x\to 3}\frac{x^2-6x+9}{x^2+x-12}=0$

3. Compute: $\lim_{x\to -\infty}\frac{5x^4-5x^3+8x^2+x}{4x^3+x^2}=$

   $\lim_{x\to -\infty}\frac{5x^4-5x^3+8x^2+x}{4x^3+x^2}=\lim_{x\to -\infty}\frac{5x^4}{4x^3}=\lim_{x\to -\infty}\frac{5x}{4}=-\infty$

   i.e., $\lim_{x\to -\infty}\frac{5x^4-5x^3+8x^2+x}{4x^3+x^2}=-\infty$

4. Compute: $\lim_{x\to -2} \frac{x^3+5}{x^2-x-6} =$

(a) 1. Try Plugging in:

$$\lim_{x\to -2} \frac{x^3+5}{x^2-x-6} = \frac{(-2)^3+5}{(-2)^2-(-2)-6} = \frac{-3}{0}$$

No Good - Zero Divide!

2. Try Factoring and Cancelling:

No Good!. "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

3. Analyze the one-sided limits:

$$\lim_{x\to -2^-} \frac{x^3+5}{x^2-x-6} = \lim_{x\to -2^-} \frac{x^3+5}{(x+2)(x-3)} = \frac{-3}{(-\varepsilon)(-5)} = \frac{-3}{(\varepsilon)(5)} = \frac{\left(-\frac{3}{5}\right)}{(\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \end{array}$$

$$\lim_{x\to -2^+} \frac{x^3+5}{x^2-x-6} = \lim_{x\to -2^+} \frac{x^3+5}{(x+2)(x-3)} = \frac{-3}{(\varepsilon)(-5)} = \frac{3}{(\varepsilon)(5)} = \frac{\left(\frac{3}{5}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to -2} \frac{x^3+5}{x^2-x-6}$ **Does Not Exist!**

5. Compute: $\lim_{x\to 5} \frac{\sqrt{44+x}-7}{x-5}$

(a) 1. Try Plugging in:

$$\lim_{x\to 5} \frac{\sqrt{44+x}-7}{x-5} = \frac{\sqrt{44+(5)}-7}{(5)-5} = \frac{0}{0}$$

No Good - Zero Divide!

2. Try Factoring and Cancelling:

$$\lim_{x\to 5} \frac{\sqrt{44+x}-7}{x-5} = \lim_{x\to 5} \frac{\sqrt{44+x}-7}{x-5} \cdot \frac{\sqrt{44+x}+7}{\sqrt{44+x}+7} = \lim_{x\to 5} \frac{\left(\sqrt{44+x}\right)^2-(7)^2}{(x-5)\left[\sqrt{44+x}+7\right]}$$

$$= \lim_{x\to 5} \frac{44+x-49}{(x-5)\left[\sqrt{44+x}+7\right]} = \lim_{x\to 5} \frac{(x-5)}{(x-5)\left[\sqrt{44+x}+7\right]} = \lim_{x\to 5} \frac{1}{\sqrt{44+x}+7}$$

$$= \frac{1}{\sqrt{44+5}+7} = \frac{1}{7+7} = \frac{1}{14}$$

i.e., $\lim_{x\to 5} \frac{\sqrt{44+x}-7}{x-5} = \frac{1}{14}$

6. Find the asymptotes and graph: $f(x) = \frac{3x^2-2}{x^2-2x-3}$

$\boxed{\text{Verticals}}$

1. Find $x$-values that cause division by zero.

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0$

$\Rightarrow x = -1$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to-1^-} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to-1^-} \frac{3x^2-2}{(x+1)(x-3)} = \frac{1}{(-\varepsilon)(-4)} = \frac{1}{(\varepsilon)(4)} = \frac{\left(\frac{1}{4}\right)}{\varepsilon} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x+1 < 0 \end{array}}$$

$$\lim_{x\to-1^+} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to-1^+} \frac{3x^2-2}{(x+1)(x-3)} = \frac{1}{(\varepsilon)(-4)} = \frac{\left(-\frac{1}{4}\right)}{\varepsilon} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x+1 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -1$ *is a vertical asymptote.*

$$\lim_{x\to3^-} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to3^-} \frac{3x^2-2}{(x+1)(x-3)} = \frac{25}{(4)(-\varepsilon)} = \frac{\left(\frac{25}{4}\right)}{(-\varepsilon)} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \end{array}}$$

$$\lim_{x\to3^+} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to3^+} \frac{3x^2-2}{(x+1)(x-3)} = \frac{25}{(4)(\varepsilon)} = \frac{\left(\frac{25}{4}\right)}{(\varepsilon)} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to-\infty} \frac{3x^2}{x^2} = \lim_{x\to-\infty} 3 = 3$

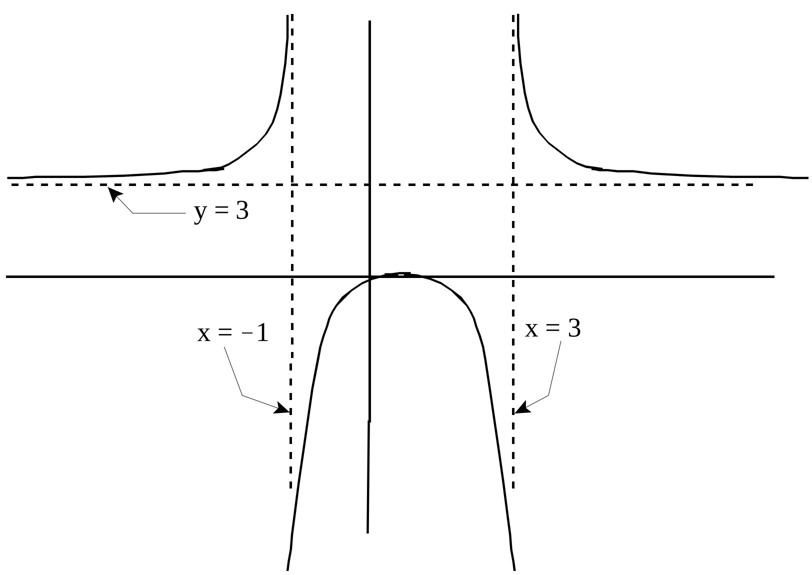
$\lim_{x\to+\infty} \frac{3x^2-2}{x^2-2x-3} = \lim_{x\to+\infty} \frac{3x^2}{x^2} = \lim_{x\to+\infty} 3 = 3$

Since the limits are finite and constant, $y = 3$ is a horizontal asymptote.

Summary:

| | |
|---|---|
| $\lim_{x\to-1^-} \frac{3x^2-2}{x^2-2x-3} = +\infty$ | |
| $\lim_{x\to-1^+} \frac{3x^2-2}{x^2-2x-3} = -\infty$ | $\lim_{x\to-\infty} \frac{3x^2-2}{x^2-2x-3} = 3$ |
| $\lim_{x\to3^-} \frac{3x^2-2}{x^2-2x-3} = -\infty$ | $\lim_{x\to+\infty} \frac{3x^2-2}{x^2-2x-3} = 3$ |
| $\lim_{x\to3^+} \frac{3x^2-2}{x^2-2x-3} = +\infty$ | |

Graph $f(x) = \frac{3x^2-2}{x^2-2x-3}$



7.

| $x =$ | $f(x) =$ |
|---|---|
| | |
| 3.5 | 15.1 |
| 3.9 | 227.8 |
| 3.99 | 1212.3 |
| 3.999 | 21156.3 |
| 3.9999 | 834561.9 |

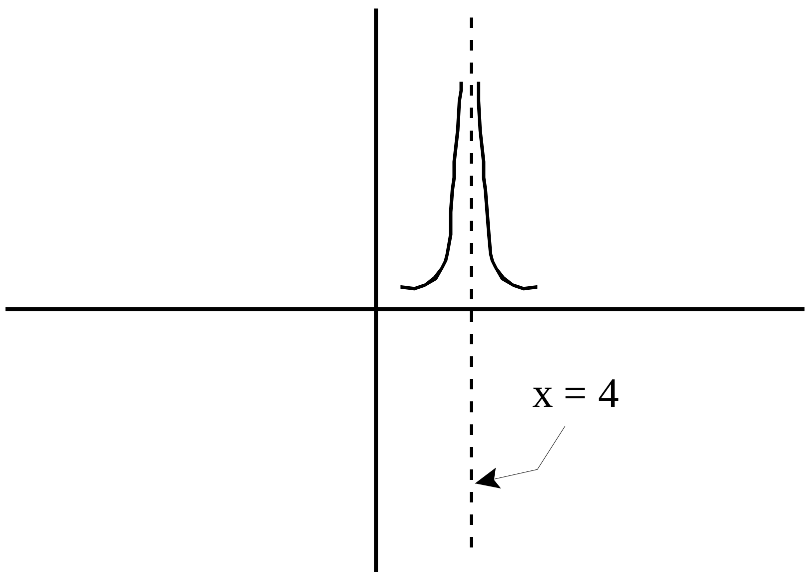
| $x =$ | $f(x) =$ |
|---|---|
| | |
| 4.5 | 15.1 |
| 4.1 | 227.8 |
| 4.01 | 1212.3 |
| 4.001 | 21156.3 |
| 4.0001 | 834561.9 |

Based on the information in the table above, do the following:

(a) $\lim_{x \to 4^-} f(x) = \infty$

(b) $\lim_{x \to 4^+} f(x) = \infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 2$. (Justify your answer.)

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{for } x \neq 2 \\ 4 & \text{for } x = 2 \end{cases}$$

If $f(x)$ is continuous at the point $x = 2$, then $\lim_{x \to 2} f(x) = f(2)$.

To see if this is true, we'll compute $\lim_{x \to 2} f(x)$.

Since the definiton of $f(x)$ changes at $x = 2$, we must compute the one-sided limits in order to determine whether the limit exists.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim_{x \to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x \to 2^-} (x+2) = 4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2-4}{x-2} = \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2} = \lim_{x \to 2^+} (x+2) = 4$

Since the one-sided limits are equal, $\lim_{x \to 4} f(x)$ exists and $\lim_{x \to 4} f(x) = 4$

In addition, note that, $4 = \lim_{x \to 2} f(x) = f(2) = 4$

i.e., $\lim_{x \to 2} f(x) = f(2)$

Hence, $f(x)$ IS continuous at $x = 2$

**EXTRA - WOW!** (7 pts) Graph $f(x)$ given the following information:

$\lim_{x \to -\infty} f(x) = +\infty$, $\lim_{x \to +\infty} f(x) = -\infty$ — No Horizontal asymptotes

$\lim_{x \to -2^-} f(x) = +\infty$, $\lim_{x \to -2^+} f(x) = -\infty$ — $x = -2$ is a vertical asymptote

$\lim_{x \to 2^-} f(x) = +\infty$, $\lim_{x \to 2^+} f(x) = -\infty$ — $x = 2$ is a vertical asymptote