Exercises Involving Real Numbers #4

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Instructions. Prove the following:

1. There exists a rational number between any two distinct real numbers.

   Corollary 1 *There exists a rational number between any two distinct rational numbers.*

   Corollary 2 *There exists a rational number between any two distinct irrational numbers.*

2. There exists an irrational number between any two distinct real numbers.

   Corollary 3 *There exists an irrational number between any two distinct rational numbers.*

   Corollary 4 *There exists an irrational number between any two distinct irrational numbers.*

3. Given a rational number z and any irrational number x, there exists an irrational number y such that $x + y = z$.

4. There exists a natural number the sum of whose digits is equal to the product of its digits.

5. A number exists whose digits have increasing values. The sum of the digits is 10 and the product of the digits is 24.

6. There exists a natural number the sum of whose digits is 10 and the product of whose digits is 36. All of the digits are prime numbers.

7. It is not known whether there are infinitely many prime pairs, i.e. odd primes whose difference is two. Examples of prime pairs are $\langle 3, 5\rangle$, $\langle 5, 7\rangle$, $\langle 11, 13\rangle$, and $\langle 71, 73\rangle$. Give three more examples of prime pairs.

8. Prove that $\langle 3, 5, 7\rangle$ is the only "prime triple". (Hint: Consider the generic prime triple, $\langle p, p+2, p+4\rangle$, and show that one of these must be divisible by 3.)