# MTH 3318 Induction Set 1b

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Pat Rossi Name ______________________________

**Instructions.** Prove the following by mathematical induction.

1. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that:

   $|x_1 + x_2 + x_3 + \ldots + x_n| \leq |x_1| + |x_2| + |x_3| + \ldots + |x_n|$ for real numbers $x_1, x_2, \ldots x_n$,

   (the General Triangle Inequality).

2. $(1+x)^n \geq 1 + nx$ for any natural number $n$ and any real number $x \geq -1$.

3. For real numbers $a, b$, with $0 \leq a \leq b$; prove that $a^n \leq b^n$.

4. $n(n+1)$ is divisible by 2 for all natural numbers, $n$.

5. Given that $\frac{d}{dx}[x^0] = 0$ and $\frac{d}{dx}[x^1] = 1$, prove that $\frac{d}{dx}[x^n] = nx^{n-1}$. You may use the product rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$.

6. $n(n+1)(n+2)$ is divisible by 6 for all natural numbers, $n$.

7. Given a set $A$ with $n$ elements, the **power set of A**, denoted $\mathbf{2}^A$, is the collection of all subsets of $A$.

   For example, given the set $A = \emptyset$, the power set of $A$ is

   $$2^A = \{\emptyset\} \ (2^A \text{ has } 2^0 = 1 \text{ element})$$

   For example, given the set $A = \{1\}$, the power set of $A$ is

   $$2^A = \{\emptyset, \{1\}\} \ (2^A \text{ has } 2^1 = 2 \text{ elements})$$

   For example, given the set $A = \{1, 2\}$, the power set of $A$ is

   $$2^A = \{\emptyset, \{1\}, \{2\}, \{1,2\}\} \ (2^A \text{ has } 2^2 = 4 \text{ elements})$$

   For example, given the set $A = \{1, 2, 3\}$, the power set of $A$ is

   $$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\} \ (2^A \text{ has } 2^3 = 8 \text{ elements})$$

   **Prove the following:** If $A$ is a set with $n$ elements, then the power set of $A$ has $2^n$ elements. (Hint: For the induction step, don't consider two unrelated sets - one with $n$ elements, and the other one with $n+1$ elements. Instead, form a set with $n+1$ elements by taking a set with $n$ elements and adding one new element to it.)