MTH 4441 Test #1

Fall 2023

Pat Rossi Name _____

1. **Define: Group**

2. **Define: Binary operation**

3. **Define:** Integers a and b **congruent modulo** n.

4. Give an alternate characterization of **congruence modulo** n.

5. **Define:** $(\mathbb{Z}_n, \oplus)$ (the **additive group of integers modulo** n)

6. **Define:** $(U_n, \odot)$ (the **multiplicative group of integers modulo** n)

7. **Prove:** If $(G, *)$ is a group, and a, b are any elements of G, then $(a * b)^{-1} = b^{-1} * a^{-1}$

8. **Define:** The **order of an element** x of a group $(G, *)$ (In your definition, specify either **additive** or **multiplicative** notation.)

9. **Prove:** The inverse of an element x in a group $(G, *)$ is unique.

10. Construct the group table for $(U_7, \odot)$

11. In the previous exercise, determine the order of the element 3

12. Construct the group table for $(\mathbb{Z}_5, \oplus)$

13. In the previous exercise, determine the order of the element 4

14. Define what it means for a binary operation $*$ to be associative.

15. Determine whether the operation $*$, given by $a * b = ab + ba$ is an associative binary operation on the set $\mathbb{R}$.

16. Fill out the group table below:

$*$	e	a	b	c
e				
a				
b				
c				