

MTH 4441 Test #1

FALL 2024

Pat Rossi

Name _____

1. **Define: Group**

2. **Define: Binary operation**

3. **Define:** Integers a and b **congruent modulo n .**

4. Give an alternate characterization of **congruence modulo n .**

5. **Define:** (\mathbb{Z}_n, \oplus) (the **additive group of integers modulo n**)

6. **Define:** (U_n, \odot) (the **multiplicative group of integers modulo n**)

7. **Prove:** If $(G, *)$ is a group, and a, b are any elements of G , then $(a * b)^{-1} = b^{-1} * a^{-1}$

8. **Define:** The **order of an element** x of a group $(G, *)$ (In your definition, specify either **additive** or **multiplicative** notation.)
9. **Prove:** The identity element e in a group $(G, *)$ is unique.
10. Construct the group table for (U_7, \odot)
11. In the previous exercise, determine the order of the element 2
12. Construct the group table for (\mathbb{Z}_6, \oplus)
13. In the previous exercise, determine the order of the element 4

14. Define what it means for a binary operation $*$ to be associative.
15. Determine whether the operation $*$, given by $a * b = ab + b$ is an associative binary operation on the set \mathbb{R} . (Justify your answer)
16. **Prove:** The inverse of an element x in a group $(G, *)$ is unique.