Proofs Involving Sets #4 (Proof by Contradiction) - Solutions

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Instructions. Prove by Contradiction

1. $\underbrace{(A \cap B) \subseteq A}_{p}$

   Proof. (By contradiction). Suppose, for the sake of contradiction, that $\underbrace{(A \cap B) \nsubseteq A}_{\sim p}$.

   $\Rightarrow \exists x \in (A \cap B)$ such that $x \notin A$.

   $\Rightarrow x \in A$ and $x \in B$ and $x \notin A$.

   In particular, $\Rightarrow \underbrace{x \in A}_{q}$ and $\underbrace{x \notin A}_{\sim q}$, a contradiction.

   Since the assumption that $(A \cap B) \nsubseteq A$ leads to a contradiction, it must be false.

   Hence, $(A \cap B) \subseteq A$. ■

2. $\underbrace{U^c = \emptyset}_{p}$

   Proof. (By contradiction). Suppose, for the sake of contradiction, that $\underbrace{U^c \neq \emptyset}_{\sim p}$.

   $\Rightarrow \exists x \in U^c$

   $\Rightarrow \underbrace{x \notin U}_{q}$

   This contradicts the definition of universe: $\underbrace{x \in U \ \forall x}_{\sim q}$.

   Since the assumption that $U^c \neq \emptyset$ leads to a contradiction, it must be false.

   Hence, $U^c = \emptyset$. ■

3. $\underbrace{(A \cap B) = \emptyset}_{\substack{\text{hypothesis} \\ p}} \Rightarrow \underbrace{A \subseteq B^c}_{\substack{\text{conclusion} \\ q}}$

Proof. (By contradiction). Let the hypothesis be given. (i.e., Suppose that $\underbrace{(A \cap B) = \emptyset}_{p}$)

Suppose, for the sake of contradiction, that $\underbrace{A \nsubseteq B^c}_{\sim q}$.

$\Rightarrow \exists x \ni x \in A$ and $x \notin B^c$

$\Rightarrow \exists x \ni x \in A$ and $x \in B$

$\Rightarrow \exists x \ni x \in (A \cap B)$

$\Rightarrow \underbrace{(A \cap B) \neq \emptyset}_{\sim p}$, but this contradicts our hypothesis, $\underbrace{(A \cap B) = \emptyset}_{p}$.

Since the assumption that $\underbrace{A \nsubseteq B^c}_{\sim q}$ leads to a contradiction, it must be false.

Hence, $\underbrace{A \subseteq B^c}_{q}$ ■