MTH 3318 - Test #2 - Solutions

Fall 2023

Pat Rossi Name ______________________________

Instructions. Fully document your work.

1. In exercises 1.a - 1.d, let p be the statement: "It is warm out," and let q be the statement: "We will go golfing." Write each statement in symbolic form.

   (a) If $\underbrace{\text{it is warm out,}}_{p}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{we will go golfing.}}_{q}$

   $p \rightarrow q$

   (b) $\underbrace{\text{It will be warm out}}_{p}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{we will not go golfing.}}_{\sim q}$

   $p \vee \sim q$

   (c) $\underbrace{\text{Being warm out}}_{p}$ $\underbrace{\text{is a necessary and sufficient condition for}}_{\leftrightarrow}$ $\underbrace{\text{me to go golfing.}}_{q}$

   $p \leftrightarrow q$

   (d) $\underbrace{\text{It will be warm out}}_{p}$ $\underbrace{\text{if}}_{\leftarrow}$ $\underbrace{\text{I go golfing.}}_{q}$

   $p \leftarrow q$ or $q \rightarrow p$

2. In exercises 2.a - 2.d, let p be the statement: "I will get a job," and let q be the statement: "I will be broke." Write each statement in words.

   (a) $p \vee q$

   $\underbrace{\text{I will get a job}}_{p}$ $\underbrace{\text{or}}_{\vee}$ $\underbrace{\text{I will be broke.}}_{q}$

   (b) $p \wedge q$

   $\underbrace{\text{I will get a job}}_{p}$ $\underbrace{\text{and}}_{\wedge}$ $\underbrace{\text{I will be broke.}}_{q}$

   (c) $p \rightarrow \sim q$

   If $\underbrace{\text{I get a job,}}_{p}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will not be broke.}}_{\sim q}$

(d) $\sim p \quad \leftrightarrow \sim q$

$\underbrace{\text{I will not get a job}}_{\sim p}\ \underbrace{\text{if and only if}}_{\leftrightarrow}\ \underbrace{\text{I am not broke.}}_{\sim q}$

3. In problems 3.a - 3.d, determine whether the given propositions are True or False:

(a) $\underbrace{\text{If } 8 > 3,}_{T}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8 > 10.}_{F}$

$T \rightarrow F = F$

(b) $\underbrace{\text{If } 8 > 3,}_{T}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8 > 5.}_{T}$

$T \rightarrow T = T$

(c) If $\underbrace{8 > 10,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{2 + 4 = 6.}_{T}$

$F \rightarrow T = T$

(d) If $\underbrace{2 + 2 = 5,}_{F}\ \underbrace{\text{then}}_{\rightarrow}\ \underbrace{8 > 10.}_{F}$

$F \rightarrow F = T$

4. In exercises 4.a-4.b construct a truth table for the statement given.

(a) $(p \vee q) \longleftrightarrow r$

p	q	r	$(p \vee q)$	$(p \vee q) \longleftrightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	T

(b) $\sim p \wedge (q \rightarrow (\sim r))$

p	q	r	$\sim p$	$\sim r$	$(q \rightarrow \tilde{\ } r)$	$\sim p \wedge (q \rightarrow (\sim r))$
T	T	T	F	F	F	F
T	T	F	F	T	T	F
T	F	T	F	F	T	F
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	T	F	T	T
F	F	F	T	T	T	T

5. For problems 5.a - 5.d, negate the given statements:

(a) All bats drink milk.

Some bats don't drink milk.

At least one bat doesn't drink milk

(b) Some dogs play poker.

No dogs play poker.

(c) No one can blow smoke rings from their ears.

Some people can blow smoke rings from their ears.

At least one person can blow smoke rings from their ears.

(d) $\exists$ a real number x, $\ni \forall$ real numbers y, $x + y = y$.

(i.e. There exists a real number x, such that for all real numbers y, $x + y = y$.)

$\sim$ ($\exists$ a real number x, $\ni \forall$ real number s y, $x + y = y$.)

$\Leftrightarrow \forall$ a real number $x, \sim$ ($\forall$ real numbers y, $x + y = y$.)

$\Leftrightarrow \forall$ a real number $x, \exists$ a real number y, $\ni \sim$ ($x + y = y$.)

$\Leftrightarrow \forall$ a real number $x, \exists$ a real number y, $\ni x + y \neq y$.

6. For problems 6.a - 6.b, disprove the given statements by providing a suitable counter-example:

   (a) If $2n$ is even, then n is also even.

   Counter-example:

   Let $n = 3$.

   Then $2n = 6$ is even, but n is odd.

   (b) If x is a factor of $(y + z)$, then x is a factor of y and x is a factor of z.

   Counter-example:

   Let $x = 2$, $y = 3$, and $y = 5$.

   Then x is a factor of $(y + z)$, but x is neither a factor of y or z.

7. In problems 7.a - 7.d, determine whether the given arguments are valid.

   (a) $(p \leftrightarrow q) \wedge (q \vee r) \ \therefore \ (p \rightarrow r)$

   Our argument is of the form:

   $$\underbrace{[(p \leftrightarrow q) \wedge (q \vee r)]}_{\text{conjunction of premises}} \rightarrow \underbrace{(p \rightarrow r)}_{\text{conclusion}}$$

p	q	r	$(p \leftrightarrow q)$	$(q \vee r)$	$(p \leftrightarrow q) \wedge (q \vee r)$	$(p \rightarrow r)$	$[(p \leftrightarrow q) \wedge (q \vee r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T

   Since the argument is not a tautology, it is INVALID.

(b) Some birds fly. All things that fly consume gasoline. Therefore, some birds consume gasoline.

Since this argument involves quantifiers (e.g., “some,” “all) we will use the Euler Circle approach.

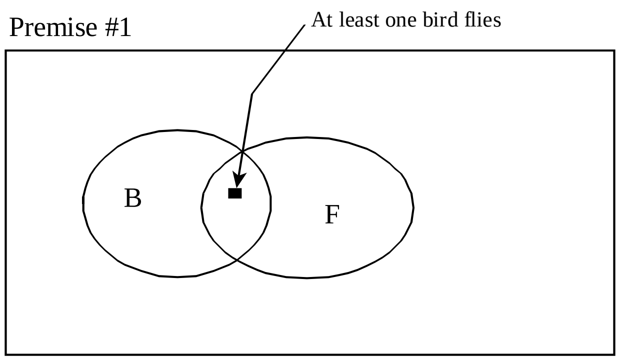
	P_1:	Some birds fly.
	P_2:	All things that fly consume gasoline.
$\therefore$	C:	Some birds consume gasoline.

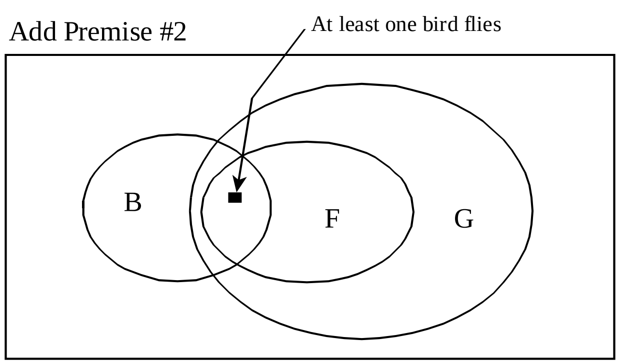
We use the following notation:

B - Birds

F - Things that fly

G - Things that consume gasoline





Since the conclusion must be true whenever the premises are true, the argument is Valid.

(c) If I shine my shoes and I comb my hair, then I will get a date. I will get a date. Therefore, if I don't shine my shoes, then I comb my hair.

p - I shine my shoes

q - I comb my hair

r - I will get a date

Premise 1: If $\underbrace{\text{I shine my shoes and I comb my hair}}_{(p\wedge q)}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I will get a date.}}_{r}$

Premise 2: $\underbrace{\text{I will get a date.}}_{r}$

Conclusion: If $\underbrace{\text{I don't shine my shoes,}}_{\sim p}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{\text{I comb my hair.}}_{q}$

Our argument has the form:

$$\underbrace{[((p \wedge q) \rightarrow r) \wedge r]}_{\text{conjunction of the premises}} \rightarrow \underbrace{(\sim p \rightarrow q)}_{\text{conclusion}}$$

p	q	r	$\sim p$	$(p \wedge q)$	$(p \wedge q) \rightarrow r$	$[((p \wedge q) \rightarrow r) \wedge r]$	$(\sim p \rightarrow q)$	$[((p \wedge q) \rightarrow r) \wedge r] \rightarrow (\sim p \rightarrow q)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	T
T	F	T	F	F	T	T	T	T
T	F	F	F	F	T	F	T	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	F	T	T	F	F
F	F	F	T	F	T	F	F	T

Since the argument is not a tautology, it is INVALID.

(d) All squares are rectangles. Some triangles are rectangles. Therefore, some squares are triangles.

Since this argument involves quantifiers (e.g., "some," "all) we will use the Euler Circle approach.

	P_1:	All squares are rectangles.
	P_2:	Some triangles are rectangles.
$\therefore$	C:	Some squares are triangles

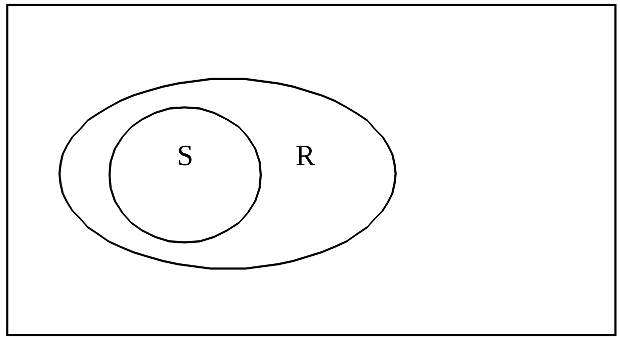
We use the following notation:

S - Squares

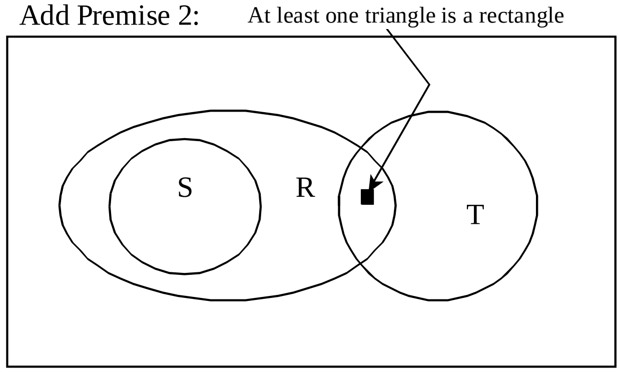
R - Rectangles

T - Triangles

Premise 1:



Add Premise 2:



Since the premises can be made True and the conclusion can be made False, the argument is INVALID.

8. Give the converse and the contrapositive of the following statement:

(a) If $\underbrace{x = 2,}_{p}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{f(x) = 5.}_{q}$

converse:

If $\underbrace{f(x) = 5,}_{q}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{x = 2.}_{p}$

contrapositive:

$\underbrace{f(x) \neq 5,}_{\sim q}$ $\underbrace{\text{then}}_{\rightarrow}$ $\underbrace{x \neq 2.}_{\sim p}$