Number Theory - Test #1

SUMMER 2023

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Instructions

Show CLEARLY how you arrive at you answers.

You can look in your text for reference (Statements of theorems, definitions, etc.)

Do not search the internet, or consult with others, for solutions (other than, perhaps, my own website)

1. State the **Well Ordering Principle**

2. State the **First Principle of Mathematical Induction** (First Principle of Finite Induction)

3. State the **Binomial Theorem**

4. State **Pascal's Rule**

5. State the **Division Algorithm**

6. Define **greatest common divisor** of a and b, denoted $\gcd(a, b)$

7. State **Divisibility Theorem 1**

8. State **Divisibility Theorem 2 (Euclid's Lemma)**

9. Define **relatively prime**

10. State **Theorem 2.2** (from our text)

11. Prove by Induction: $1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$

    i.e., $\sum_{i=1}^{n} (4i - 3) = 2n^2 - n$

12. Prove by Induction: $2 \cdot 6 \cdot 10 \cdot 14 \cdot \ldots \cdot (4n - 2) = \frac{(2n)!}{n!}$

    i.e., $\prod_{i=1}^{n} (4i - 2) = \frac{(2n)!}{n!}$

13. Prove: $\binom{n}{0}3^n - \binom{n}{1}3^{n-1} + \binom{n}{2}3^{n-2} - \binom{n}{3}3^{n-3} + \ldots + (-1)^n = 2^n$

14. Show, algebraically and with "dot diagrams," that $2o_n + 2s_n = o_{2n+1}$

15. Show, algebraically and with "dot diagrams," that $o_n + o_{n+1} + 2s_{n+1} = s_{2n+2}$

16. Prove that the cube of a natural number cannot be of the form $4n + 2$

17. Prove that for any integer a, $\gcd(5a + 2, 7a + 3) = 1$