MTH 1125 12pm Class - Test #4 - Solutions

FALL 2020

Pat Rossi   Name ______________________

Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(12x^3 + 6x^2 - 12x + 2\sqrt{x} + 4\right) dx =$

$\int \left(12x^3 + 6x^2 - 12x + 2\sqrt{x} + 4\right) dx = \int \left(12x^3 + 6x^2 - 12x + 2x^{\frac{1}{2}} + 4\right) dx$

$= 12\left[\frac{x^4}{4}\right] + 6\left[\frac{x^3}{3}\right] - 12\left[\frac{x^2}{2}\right] + 2\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + 4x + C = 3x^4 + 2x^3 - 6x^2 + \frac{4}{3}x^{\frac{3}{2}} + 4x + C$

i.e., $\int \left(12x^3 + 6x^2 - 12x + 2\sqrt{x} + 4\right) dx = 3x^4 + 2x^3 - 6x^2 + \frac{4}{3}x^{\frac{3}{2}} + 4x + C$

2. **Compute:** $\int \left(3x^2+8x+5\right)^9 (3x+4)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(3x^2+8x+5\right)^9$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(3x^2+8x+5\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^2+8x+5\right)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+4)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(3x^2+8x+5\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 3x^2+8x+5 \\ \Rightarrow \frac{du}{dx} &= 6x+8 \\ \Rightarrow du &= (6x+8)\,dx \\ \Rightarrow \frac{1}{2}du &= (3x+4)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(3x^2+8x+5\right)^9}_{u^9} \underbrace{(3x+4)\,dx}_{\frac{1}{2}du} = \int u^9 \tfrac{1}{2}du = \tfrac{1}{2}\int u^9 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{2}\int u^9 du = \tfrac{1}{2}\left[\tfrac{u^{10}}{10}\right] + C = \tfrac{1}{20}u^{10} + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(3x^2+8x+5\right)^9 (3x+4)\,dx = \underbrace{\frac{1}{20}\left(3x^2+8x+5\right)^{10} + C}_{\frac{1}{20}u^{10}+C}$$

i.e., $\int \left(3x^2+8x+5\right)^9 (3x+4)\,dx = \frac{1}{20}\left(3x^2+8x+5\right)^{10} + C$

3. **Compute:** $\int \left(2\sin(x) - 5\sec^2(x) + 2\csc(x)\cot(x)\right) dx =$

$\int \left(2\sin(x) - 5\sec^2(x) + 2\csc(x)\cot(x)\right) dx = 2\left[-\cos(x)\right] - 5\left[\tan(x)\right] + 2\left[-\csc(x)\right] + C$

$= -2\cos(x) - 5\tan(x) - 2\csc(x) + C$

i.e., $\int \left(2\sin(x) - 5\sec^2(x) + 2\csc(x)\cot(x)\right) dx = -2\cos(x) - 5\tan(x) - 2\csc(x) + C$

4. **Compute:** $\int \cos\left(5x^2+6x+3\right)(5x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(5x^2+6x+3\right)$ with outer = $\cos$, inner = $5x^2+6x+3$

   outer ↗ ↖ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(5x^2+6x+3\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(5x^2+6x+3\right)}_{\text{function}} - - - - \rightarrow \underbrace{(5x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(5x^2+6x+3\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 5x^2+6x+3 \\ \Rightarrow \frac{du}{dx} &= 10x+6 \\ \Rightarrow du &= (10x+6)\,dx \\ \Rightarrow \frac{1}{2}du &= (5x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(5x^2+6x+3\right)}_{\cos(u)}\underbrace{(5x+3)\,dx}_{\frac{1}{2}du} = \int \cos(u)\ \frac{1}{2}du = \frac{1}{2}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\left[\sin(u)\right] + C = \frac{1}{2}\sin(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \cos\left(5x^2+6x+3\right)(5x+3)\,dx = \underbrace{\frac{1}{2}\sin\left(5x^2+6x+3\right)+C}_{\frac{1}{2}\sin(u)+C}$$

i.e., $\int \cos\left(5x^2+6x+3\right)(5x+3)\,dx = \frac{1}{2}\sin\left(5x^2+6x+3\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(6x^2+4x+2\right)dx=$

$$\int_{-1}^{1}\underbrace{\left(6x^2+4x+2\right)}_{f(x)}dx=\underbrace{\left[6\frac{x^3}{3}+4\frac{x2}{2}+2x\right]_{-1}^{1}}_{F(x)}=\underbrace{\left[2x^3+2x^2+2x\right]_{-1}^{1}}_{F(x)}=$$

$$=\underbrace{\left[2\left(1\right)^3+2\left(1\right)^2+2\left(1\right)\right]}_{F(1)}-\underbrace{\left[2\left(-1\right)^3+2\left(-1\right)^2+2\left(-1\right)\right]}_{F(-1)}=6-(-2)=8$$

$$\boxed{\text{i.e., }\int_{-1}^{1}\left(6x^2+4x+2\right)dx=8}$$

6. **Compute:** $\int_{-1}^{1} \left(x^3+1\right)^4 x^2 \; dx =$ (The answer may not be a whole number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(x^3+1\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(x^3+1\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(x^3+1\right)}_{\text{function}} - - - - - \rightarrow \underbrace{x^2}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(x^3+1\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= x^3+1 \\ \Rightarrow \frac{du}{dx} &= 3x^2 \\ \Rightarrow du &= 3x^2 \; dx \\ \Rightarrow \frac{1}{3}du &= x^2 \; dx \end{aligned}$$

When $x=-1$, $u = x^3+1 = (-1)^3+1 = 0$
When $x=1$, $u = x^3+1 = (1)^3+1 = 2$

3. Analyze in terms of u and du

$$\int_{x=-1}^{x=1} \underbrace{\left(x^3+1\right)^4}_{u^4} \underbrace{x^2 \; dx}_{\frac{1}{3}du} = \int_{u=0}^{u=2} u^4 \cdot \frac{1}{3}du = \frac{1}{3}\int_{u=0}^{u=2} u^4 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{3}\int_{u=0}^{u=2} u^4 du = \frac{1}{3}\left[\frac{u^5}{5}\right]_{u=0}^{u=2} = \left[\frac{u^5}{15}\right]_{u=0}^{u=2} = \underbrace{\frac{(2)^5}{15}}_{F(2)} - \underbrace{\frac{(0)^5}{15}}_{F(0)} = \frac{32}{15} - \left(\frac{0}{15}\right) = \frac{32}{15}$$

i.e., $\int_{x=-1}^{x=1} \left(x^3+1\right)^4 x^2 \; dx = \frac{32}{15}$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(2x^4+3x^2\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(2x^4+3x^2\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{2x^4+3x^2}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(8x^3+6x\right)}_{g'(x)} = \frac{8x^3+6x}{2x^4+3x^2}$$

i.e., $\frac{d}{dx}\left[\ln\left(2x^4+3x^2\right)\right] = \frac{8x^3+6x}{2x^4+3x^2}$

8. **Compute:** $\int \frac{2x+1}{3x^2+3x+5}dx =$

$$\int \frac{2x+1}{3x^2+3x+5}dx \underbrace{=}_{\text{re-write}} \int \frac{1}{3x^2+3x+5}\ (2x+1)\ dx$$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\frac{1}{3x^2+3x+5}$ is the same as $\left(3x^2+3x+5\right)^{-1}$, so it is a function raised to a power.

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = 3x^2+3x+5$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^2+3x+5\right)}_{\text{function}} - - - - \rightarrow \underbrace{(2x+1)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = 3x^2+3x+5$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 3x^2+3x+5 \\ \Rightarrow \frac{du}{dx} &= 6x+3 \\ \Rightarrow du &= (6x+3)\,dx \\ \Rightarrow \frac{1}{3}du &= (2x+1)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\underbrace{\int \frac{1}{3x^2+3x+5}}_{\frac{1}{u}} \underbrace{(2x+1)\ dx}_{\frac{1}{3}du} = \int \frac{1}{u}\frac{1}{3}du = \frac{1}{3}\int \frac{1}{u}du$$

4. Integrate (in terms of u).

$$\frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln|u| + C$$

5. Re-express in terms of the original variable, x.

$$\int \frac{2x+1}{3x^2+3x+5}dx = \underbrace{\frac{1}{3}\ln\left|3x^2+3x+5\right| + C}_{\frac{1}{3}\ln|u|+C}$$

i.e., $\int \frac{2x+1}{3x^2+3x+5} dx = \frac{1}{3} \ln \left|3x^2 + 3x + 5\right| + C$