MTH 1126 Test #3 - Solutions

Spring 2004

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Instructions. Show clearly how you arrive at your answers.

1. Compute: $\int \sin^4(x)\cos^3(x)\,dx =$

   Pull out a factor of $\cos(x)$ to serve as the "future du."

   $= \int \sin^4(x)\cos^2(x)\underbrace{\cos(x)\,dx}_{\text{future } du}$

   Note: This means that $u = \sin(x)$

   Convert the remaining cosines into sines, using $\cos^2(x) = 1 - \sin^2(x)$.

   $= \int \underbrace{\sin^4(x)}_{u^4}\underbrace{\left(1-\sin^2(x)\right)}_{1-u^2}\underbrace{\cos(x)\,dx}_{\text{future } du}$

   $= \int (u^4)(1-u^2)\,du = \int (u^4 - u^6)\,du = \frac{u^5}{5} - \frac{u^7}{7} + C = \frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C$

   i.e., $\int \sin^4(x)\cos^3(x)\,dx = \frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C$

2. Compute: $\int \sec^4(x)\tan^4(x)\,dx =$

   Pull out a factor of $\sec^2(x)$ to serve as the "future du."

   $= \int \sec^2(x)\tan^4(x)\underbrace{\sec^2(x)}_{\text{future } du}dx$

   Note: This means that $u = \tan(x)$

   Convert the remaining secants into tangents, using $\tan^2(x) + 1 = \sec^2(x)$.

   $= \int \underbrace{\left(\tan^2(x)+1\right)}_{u^2+1}\underbrace{\tan^4(x)}_{u^4}\underbrace{\sec^2(x)}_{\text{future } du}dx$

   $= \int (u^2+1)(u^4)\,du = \int (u^6+u^4)\,du = \frac{u^7}{7} + \frac{u^5}{5} + C = \frac{\tan^7(x)}{7} + \frac{\tan^5(x)}{5} + C$

   i.e., $\int \sec^4(x)\tan^4(x)\,dx = \frac{\tan^7(x)}{7} + \frac{\tan^5(x)}{5} + C$

3. Compute: $\int \frac{x}{x^2+x-6}dx =$

   Observe: $\frac{x}{x^2+x-6} = \frac{x}{(x+3)(x-2)}$. So decompose this quotient.

$\frac{x}{x^2+x-6} = \frac{x}{(x+3)(x-2)} = \frac{c_1}{(x+3)} + \frac{c_2}{(x-2)}$

$\Rightarrow x = c_1(x-2) + c_2(x+3)$

$\boxed{\text{Let } x = 2}$

$\Rightarrow 2 = c_2((2)+3)$

$\Rightarrow 2 = 5c_2$

$\Rightarrow c_2 = \frac{2}{5}$

$\boxed{\text{Let } x = -3}$

$\Rightarrow -3 = c_1((-3)-2)$

$\Rightarrow -3 = -5c_1$

$\Rightarrow c_1 = \frac{3}{5}$

$\Rightarrow \int \frac{x}{x^2+x-6}dx = \int \left(\frac{c_1}{(x+3)} + \frac{c_2}{(x-2)}\right)dx = \int \frac{\frac{3}{5}}{(x+3)}dx + \int \frac{\frac{2}{5}}{(x-2)}dx = \frac{3}{5}\int \frac{1}{(x+3)}dx + \frac{2}{5}\int \frac{1}{(x-2)}dx$

$= \frac{3}{5}\ln|x+3| + \frac{2}{5}\ln|x-2| + C$

i.e., $\int \frac{x}{x^2+x-6}dx = \frac{3}{5}\ln|x+3| + \frac{2}{5}\ln|x-2| + C$

4. Compute: $\int x\ln(x)\,dx =$

$$\boxed{\begin{array}{ll} u = \ln(x) & dv = xdx \\ \frac{du}{dx} = \frac{1}{x} & \int dv = \int xdx \\ du = \frac{1}{x}dx & v = \frac{1}{2}x^2 \end{array}}$$

$\int x\ln(x)\,dx = \int \underbrace{\ln(x)}_{u}\underbrace{xdx}_{dv} = uv - \int vdu = (\ln(x))\left(\frac{1}{2}x^2\right) - \int \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}dx\right)$

$= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int xdx = \frac{1}{2}x^2\ln(x) - \frac{1}{2}\left(\frac{x^2}{2}\right) + C = \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C$

i.e., $\int x\ln(x)\,dx = \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C$

5. Compute: $\int \frac{1}{9+4x^2}dx =$

Try to fit this to the form: $\int \frac{1}{a^2+u^2}du$

$$\begin{aligned}
a^2 &= 9 \\
a &= 3 \\
u^2 &= 4x^2 \\
u &= 2x \\
\tfrac{du}{dx} &= 2 \\
du &= 2dx \\
\tfrac{1}{2}du &= dx
\end{aligned}$$

$\int \frac{1}{9+4x^2}dx = \int \frac{1}{a^2+u^2}\frac{1}{2}dx = \frac{1}{2}\int \frac{1}{a^2+u^2}dx = \frac{1}{2}\left[\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C\right] = \frac{1}{2}\left[\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right) + C\right]$
$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + C$

i.e., $\int \frac{1}{9+4x^2}dx = \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + C$

6. Compute: $\int \frac{1}{\sqrt{4-9x^2}}dx =$

   Try to fit this to the form: $\int \frac{1}{\sqrt{a^2-u^2}}du$

$$\begin{aligned}
a^2 &= 4 \\
a &= 2 \\
u^2 &= 9x^2 \\
u &= 3x \\
\tfrac{du}{dx} &= 3 \\
du &= 3dx \\
\tfrac{1}{3}du &= dx
\end{aligned}$$

$\int \frac{1}{\sqrt{4-9x^2}}dx = \int \frac{1}{\sqrt{a^2-u^2}}\frac{1}{3}du = \frac{1}{3}\int \frac{1}{\sqrt{a^2-u^2}}du = \frac{1}{3}\sin^{-1}\left(\frac{u}{a}\right) + C = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$

i.e., $\int \frac{1}{\sqrt{4-9x^2}}dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$