

Exercises Involving Real Numbers #1

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Name _____

Instructions. Prove:

1. For all real numbers a, b, c ; $a < b \Leftrightarrow a + c < b + c$
2. For all real numbers a, b, c ; $a < b \Leftrightarrow a - c < b - c$
3. For all real numbers a, b, c with $c > 0$; $a < b \Leftrightarrow ac < bc$
4. For all real numbers a, b, c with $c > 0$; $a < b \Leftrightarrow \frac{a}{c} < \frac{b}{c}$
5. For all real numbers a, b, c with $c < 0$; $a < b \Leftrightarrow ac > bc$
6. For all real numbers a, b, c with $c < 0$; $a < b \Leftrightarrow \frac{a}{c} > \frac{b}{c}$
7. $0 < r < |x| \Leftrightarrow x < -r$ or $r < x$
8. $0 \leq |x| < r \Leftrightarrow -r < x < r$
9. Prove or disprove:
 - (a) $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow x + y \in \mathbf{Q}$
 - (b) $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow x - y \in \mathbf{Q}$
 - (c) $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow xy \in \mathbf{Q}$
 - (d) $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow \frac{x}{y} \in \mathbf{Q}$
 - (e) Under what conditions does $x \in \mathbf{Q}$ and $y \in \mathbf{Q} \Rightarrow \frac{x}{y} \in \mathbf{Q}$?
 - (f) $x \in \mathbf{Q}$ and $y \in \mathbf{Q}^c \Rightarrow x + y \in \mathbf{Q}^c$
 - (g) $x \in \mathbf{Q}^c$ and $y \in \mathbf{Q}^c \Rightarrow x + y \in \mathbf{Q}^c$
 - (h) $x \in \mathbf{Q}$ and $y \in \mathbf{Q}^c \Rightarrow xy \in \mathbf{Q}^c$
 - (i) $x \in \mathbf{Q}^c$ and $y \in \mathbf{Q}^c \Rightarrow xy \in \mathbf{Q}^c$