MTH 3318 Test #1

Spring 2013

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Instructions. Fully document your work.

For problems 1 - 2, prove one using Mathematical Induction:

1. For $0 \leq a \leq b$; prove that $a^n \leq b^n$.

2. Given that $\frac{d}{dx}\left[x^0\right] = 0$ and $\frac{d}{dx}\left[x^1\right] = 1$, prove that $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$. You may use the product rule: $\frac{d}{dx}\left[f(x)\,g(x)\right] = f'(x)\,g(x) + g'(x)\,f(x)$.

For problems 3 - 4, prove one using Mathematical Induction:

3. $(1+x)^n \geq 1 + nx$ for any natural number n and any real number $x \geq -1$.

4. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that: $|x_1 + x_2 + x_3 + \ldots + x_n| \leq |x_1| + |x_2| + |x_3| + \ldots + |x_n|$ (the General Triangle Inequality).

For problems 5- 10 prove three using Mathematical Induction.

5. $1 + 3 + 5 + \ldots + (2n-1) = n^2$
   i.e. $\sum_{i=1}^{n} (2i-1) = n^2$

6. $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$
   i.e. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

7. $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
   i.e. $\sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)} = \frac{n}{2n+1}$

8. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$
   i.e. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

9. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \ldots + n^3$ all natural numbers, n.

10. $\frac{n^4}{4} < 1^3 + 2^3 + 3^3 + \ldots + n^3$ all natural numbers, n.