Exercises Involving Real Numbers #3

FALL 2005

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Instructions. Disprove by providing a counter-example.

1. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$
2. $\sum_{i=1}^{n} x_i^2 = \left(\sum_{i=1}^{n} x_i\right)^2$
3. $x \leq x^2$
4. $xy \geq x + y$
5. $\frac{1}{x} \leq x$
6. $\frac{1}{x} + \frac{1}{y} \geq \frac{1}{x+y}$
7. $\frac{2}{\frac{1}{x}+\frac{1}{y}} \leq \frac{x+y}{2}$
8. $|x + y| = |x| + |y|$
9. $\sqrt{xy} \leq \frac{(x+y)}{2}$
10. $xy \leq x\,|y|$
11. $\sum_{i=1}^{n} x_i^2 \geq \sum_{i=1}^{n} x_i$
12. $\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2 \leq \sum_{i=1}^{n} (x_i + y_i)^2$
13. $\sqrt{x} \leq x$, for all $x \geq 0$.
14. $|\log x| \leq x$, for all $x > 0$.
15. If $x < y$, then $\frac{1}{x} > \frac{1}{y}$ $(x, y \neq 0)$
16. If $x \leq y$, then $\frac{y}{x} \geq 1$ $(x \neq 0)$
17. If p is a prime number, then $p + 1$ is not prime.
18. If p is a prime number, then $p^2 + 1$ is not prime.
19. (Johnsonbaugh) For any prime p, $n^2 + n + p$ is prime for $n = 0, 1, \ldots, p - 1$.