MTH 6610 Reading Assignment #7 - Answers

SUMMER 2018

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Instructions.

Read pages 213-237 and answer the following questions.

1. What act caused an end to mathematical study in Alexandria? When?

   After Ptolemy VII ascended to power in 146 B.C., he banished from Egypt all scientists and scholars who had not demonstrated loyalty to him during his quest for the throne.

2. What noteworthy contributions did the Romans make to mathematics? (Somewhat of a trick question)

   From 750 B.C. to 450 A.D., there appeared no Roman Mathematician of note. **They made no known noteworthy contribution to mathematics.** Their only concern was the application of mathematics to engineering projects (viaducts, bridges, roads, public buildings, land surveys) without having any grasp of the theory behind the mathematics that they applied.

3. What development effectively put an end to the study of science and mathematics in the Western Roman Empire?

   In 392 A. D. , Emperor Theodosius, a Christian, put laws into effect that closed all pagan temples in the empire and forbade the exercise of pagan exercises of any kind – even those conducted in the privacy of one's own home. This attitude of "religious intolerance" was mirrored by masses of Christians themselves, who viewed the Bible as the source of all knowledge and, with great contempt, viewed science and mathematics as being pagan. In a dismaying case of 'role reversal,' Christians became the persecutors of Greek (Alexandrian) scholars. With the encouragement of the Roman emperors, they looted the Museum and the Library. (In 641 A.D., what little that was left over from the Christian riots was destroyed by the Moslems who viewed the writings as blasphemy against the Koran.)

4. How is it that the accumulated knowledge of Greek/Alexandrian mathematicians was preserved over the centuries for rediscovery by Renaissance mathematicians?

   Byzantine scholars, although not actively engaged in original research, were actively engaged in preserving and multiplying the available copies of "Greek" manuscripts.

5. How could we describe Diophantus' main contribution to mathematics, and what is the name of the "text" that contains his work in this area?

   Diophantus' text was called Artihmetica – an assortment of 189 problems, along with their solutions.

   Diophantus' main contribution, referred to as "Syncopated algebra," was a system of algebraic shorthand, used to denote such things as exponentiation (up to the sixth power), equality, subtraction, and reciprocals.

Remark: Once again, note that something as seemingly arbitrary as notation would turn out to be so indispensable in removing roadblocks and enabling mathematicians to make great gains. Hence, there are sound reasons why our students should do things "our way" or "the conventional way," instead of doing things "their way." Armed with this fact, we may want to encourage our students not to let lack of good notation prevent *them* from making giant strides in mathematics!

6. What is Diophantus' supposed background?

   He lived in Alexandria c. 250 A.D. and was most likely a "Hellenized" Babylonian.

7. For what riddle is Diophantus well known?

   Hi boyhood lasted for $\frac{1}{6}$ of his life; his beard grew after $\frac{1}{12}$ more; after $\frac{1}{7}$ more, he married; and his son was born 5 years later; the son lived to $\frac{1}{2}$ his father's age and the father died 4 years after his son. Letting x be the age at which Diophantus died, we have the equation:

$$\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$$

8. What is noteworthy about the solutions to Diophantus' equations?

   Only positive rational solutions were admitted and he was satisfied upon finding a single solution, even though the equation may have had more than one solution.

   He considered negative solutions to be "absurd" and "impossible."

Remark: Apparently, Diophantus' abhorrence toward negative solutions to an equation sprung from the fact that, while he could conceive of the **reality** of positive *rational* solutions (e.g., $\frac{7}{8}$ of an apple or $\frac{2}{3}$ of a pie made perfect sense to him), he could not conceive of the **reality** of *negative* solutions (e.g., how on earth can you have -1 apple or $-\frac{3}{4}$ of a pie???).

Today, we can readily conceive of the reality of negative solutions to an equation. (e.g., a negative balance on our checking account means that we're overdrawn. A negative elevation means that we're standing below sea level. A negative velocity means that an object is moving backwards.)

However, we may fall victim to the same kind of "tunnel vision" that Diophantus had in another way. We've all heard of, and used, "imaginary numbers" and "complex numbers." The very name "imaginary numbers" quite naturally leads us to think that these numbers "don't really exist," and hence, we may not be convinced of the **reality** of *imaginary* or *complex* solutions.

While the notions of: $2 + 3i$ apples; standing at an elevation of $180 - 4i$ feet; and travelling at a velocity of $60 + 15i$ miles per hour; may rightly seem absurd, this does not mean that imaginary and complex solutions don't, in some cases, represent "real world" solutions any more than negative solutions don't, in some cases, represent "real world" solutions.

Imaginary and complex numbers can be used to realistically model things like electronic circuitry, fluid dynamics, and air flow. In these settings, imaginary and complex solutions DO represent "real world" solutions.

9. What mathematicians are credited with the earliest attempts to systematically solve the indeterminate equation

$$ax + by = c$$

by a general method?

Hindu Mathematicians Aryabhata (476-?), Brahmagupta (c. 600), Mahavira (c. 850), and Bhaskara (1114-1185).

10. What influence did early Indian mathematicians "receive"?

When Alexander the Great invaded India, he established Greek colonies in India and on its borders. As he did everywhere he went, he established a Hellenistic culture in these colonies and their continued existence served as a conduit of information and ideas between Asia and the Mediterranean world.

Much later, the Chinese were also to exert mathematical influence on India.

11. What contributions to mathematics are credited to Aryabhata?

Aryabhata investigated the summation of arithmetic and geometric series, constructed a table of sines of angles in the first quadrant, studied first and second degree indeterminate equations (i.e., equations for which there are infinitely many solutions) and calculated the value of pi accurately to 4 decimal places ($\pi = 3.1416$).

12. What contributions to mathematics are credited to Brahmagupta?

Brahmagupta introduced negative numbers, formulated a rule for obtaining both roots of a quadratic equation, and derived the formula

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

for the area of a cyclic quadrilateral whose sides are of length a, b, c, and d and whose "semi-perimeter" is s. (A cyclic quadrilateral is a four-sided polygon whose vertices all lie on the circumference of a circle.)

He was the first to obtain all integral solutions to the indeterminate, Diophantine equation $ax + by = c$.

13. What exactly IS a "Diophantine Equation"?

A Diophantine Equation is an equation in one or more unknowns that admits only integer solutions.

14. By whom were simultaneous Diophantine Equations first studied, and when?

In the 1^{st} Century, by Chinese mathematician Sun-Tsu

15. For what theorem is Pappus known?

Referring to "solids of revolution" (i.e., a solid defined by revolving a region in the plane about an axis of revolution), proved the following: The volume is equal to the product of the area of the plane region times the distance traversed by of the center of gravity of the region as it is revolved about the axis of revolution.

16. What is considered to be the main value of Pappus' writings?

    It preserved the results and observations of other mathematicians that would otherwise have been lost for all time. (Particularly the missing books of Euclid and Apollonius)

17. What "traumas" were inflicted upon the Alexandrian Library, and by whom?

    Christian mobs murdered many of the Museum's scholars in the streets of Alexandria, including Hypatia, daughter of Theon of Alexandria. In 641 A.D., what little that was left over from the Christian riots was destroyed by the Moslems who viewed the writings as blasphemy against the Koran. In addition to those "traumas" already mentioned, in question 3, it turns out that Julius Caesar played a role in destroying many manuscripts in his siege of Alexandria in 48 B.C. He had the Egyptian fleet set afire and the conflagration spread to the Library – consuming the entire building.