MTH 1125 12pm Class - Test #4 -Solutions

Fall 2022

Pat Rossi Name ______________________________

Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(16x^3 + 15x^2 - 12x + 7 + 12\sqrt{x}\right) dx = \int \left(16x^3 + 15x^2 - 12x + 7 + 12x^{\frac{1}{2}}\right) dx$

$= 16\left[\frac{x^4}{4}\right] + 15\left[\frac{x^3}{3}\right] - 12\left[\frac{x^2}{2}\right] + 7x + 12\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + C$

$= 4x^4 + 5x^3 - 6x^2 + 7x + 12\left[\left(\frac{2}{3}\right)x^{\frac{3}{2}}\right] + C = 4x^4 + 5x^3 - 6x^2 + 7x + 8x^{\frac{3}{2}} + C$

i.e., $\int \left(16x^3 + 15x^2 - 12x + 7 + 12\sqrt{x}\right) dx = 4x^4 + 5x^3 - 6x^2 + 7x + 8x^{\frac{3}{2}} + C$

2. **Compute:** $\int (8x^2+12x+5)^4 (4x+3)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $(8x^2+12x+5)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (8x^2+12x+5)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(8x^2+12x+5)}_{\text{function}} - - - - \rightarrow \underbrace{(4x+3)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (8x^2+12x+5)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 8x^2+12x+5 \\ \Rightarrow \frac{du}{dx} &= 16x+12 \\ \Rightarrow du &= (16x+12)\,dx \\ \Rightarrow \tfrac{1}{4}du &= (4x+3)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{(8x^2+12x+5)^4}_{u^4}\underbrace{(4x+3)\,dx}_{\frac{1}{4}du} = \int u^4 \tfrac{1}{4}du = \tfrac{1}{4}\int u^4 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4}\int u^4 du = \tfrac{1}{4}\left[\tfrac{u^5}{5}\right] + C = \tfrac{1}{20}u^5 + C$$

5. Re-express in terms of the original variable, x.

$$\int (8x^2+12x+5)^4 (4x+3)\,dx = \underbrace{\frac{1}{20}(8x^2+12x+5)^5 + C}_{\frac{1}{20}u^5+C}$$

i.e., $\int \left(8x^2+12x+5\right)^4 (4x+3)\,dx = \frac{1}{20}\left(8x^2+12x+5\right)^5 + C$

3. **Compute:** $\int \left(9\cos(x) - 6\sec^2(x) + 2\sec(x)\tan(x)\right) dx =$

$\int \left(9\cos(x) - 6\sec^2(x) + 2\sec(x)\tan(x)\right) dx = 9\left[\sin(x)\right] - 6\left[\tan(x)\right] + 2\left[\sec(x)\right] + C$

$= 9\sin(x) - 6\tan(x) + 2\sec(x) + C$

i.e., $\int \left(9\cos(x) - 6\sec^2(x) + 2\sec(x)\tan(x)\right) dx = 9\sin(x) - 6\tan(x) + 2\sec(x) + C$

4. **Compute:** $\int \sin(6x^2+20x+3)(3x+5)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\sin(6x^2+20x+3)$ — outer: $\sin$, inner: $(6x^2+20x+3)$

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (6x^2+20x+3)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(6x^2+20x+3)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+5)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (6x^2+20x+3)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+20x+3 \\ \Rightarrow \frac{du}{dx} &= 12x+20 \\ \Rightarrow du &= (12x+20)\,dx \\ \Rightarrow \tfrac{1}{4}du &= (3x+5)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\sin\left(6x^2+20x+3\right)}_{\sin(u)}\underbrace{(3x+5)\,dx}_{\frac{1}{4}du} = \int \sin(u)\,\tfrac{1}{4}du = \tfrac{1}{4}\int \sin(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4}\int \sin(u)\,du = \tfrac{1}{4}\left[-\cos(u)\right] + C = -\tfrac{1}{4}\cos(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \sin(6x^2+20x+3)(3x+5)\,dx = \underbrace{-\frac{1}{4}\cos\left(6x^2+20x+3\right)+C}_{-\frac{1}{4}\cos(u)+C}$$

i.e., $\int \sin\left(6x^2+20x+3\right)(3x+5)\,dx = -\frac{1}{4}\cos\left(6x^2+20x+3\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(3x^2+4x+2\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(3x^2+4x+2\right)}_{f(x)}dx = \underbrace{\left[3\frac{x^3}{3}+4\frac{x2}{2}+2x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[x^3+2x^2+2x\right]_{-1}^{1}}$$

$$= \underbrace{\left[(1)^3+2\,(1)^2+2\,(1)\right]}_{F(1)} - \underbrace{\left[(-1)^3+2\,(-1)^2+2\,(-1)\right]}_{F(-1)}$$

$$= 5-(-1) = 6$$

i.e., $\int_{-1}^{1}\left(3x^2+4x+2\right)dx = 6$

6. **Compute:** $\int_{x=0}^{x=1} (2x^2+1)^4 \, x\,dx =$ (The answer may not be a whole number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $(2x^2+1)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = (2x^2+1)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{(2x^2+1)}_{\text{function}} - - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = (2x^2+1)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 2x^2+1 \\ \Rightarrow \tfrac{du}{dx} &= 4x \\ \Rightarrow du &= 4x\ dx \\ \Rightarrow \tfrac{1}{4}du &= x\ dx \end{aligned}$$

   When $x=0$, $u = 2x^2+1 = 2(0)^2+1 = 1$
   When $x=1$, $u = 2x^2+1 = 2(1)^2+1 = 3$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{(2x^2+1)^4}_{u^4} \underbrace{x\ dx}_{\frac{1}{4}du} = \int_{u=1}^{u=3} u^4 \cdot \tfrac{1}{4}du = \tfrac{1}{4}\int_{u=1}^{u=3} u^4 du$$

   Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\tfrac{1}{4}\int_{u=1}^{u=3} u^4 du = \tfrac{1}{4}\left[\tfrac{u^5}{5}\right]_{u=1}^{u=3} = \left[\tfrac{u^5}{20}\right]_{u=1}^{u=3} = \underbrace{\frac{(3)^5}{20}}_{F(3)} - \underbrace{\frac{(1)^5}{20}}_{F(1)} = \tfrac{243}{20} - \left(\tfrac{1}{20}\right) = \tfrac{242}{20} = \tfrac{121}{10}$$

$$\text{i.e., } \int_{x=0}^{x=1} \left(2x^2+1\right)^4 x \; dx = \tfrac{121}{10}$$ $\int_0^1 \left(2x^2+1\right)^4 x \; dx = \frac{121}{10}$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(12x^3+18x^2-5\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(12x^3+18x^2-5\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{12x^3+18x^2-5}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(36x^2+36x\right)}_{g'(x)} = \frac{36x^2+36x}{12x^3+18x^2-5}$$

$$\text{i.e., } \frac{d}{dx}\left[\ln\left(12x^3+18x^2-5\right)\right] = \frac{36x^2+36x}{12x^3+18x^2-5}$$

Extra! (Wow! - 5 pts. Can you believe it?) **Compute:** $\frac{d}{dx}\left[\ln\left(\sqrt{\frac{12x^4-4x^2+2}{6x^3-5x}}\right)\right] ==$

$$\frac{d}{dx}\left[\ln\left(\sqrt{\frac{12x^4-4x^2+2}{6x^3-5x}}\right)\right] \underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{d}{dx}\left[\ln\left[\left(\frac{12x^4-4x^2+2}{6x^3-5x}\right)^{\frac{1}{2}}\right]\right] \underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{d}{dx}\left[\frac{1}{2}\ln\left(\frac{12x^4-4x^2+2}{6x^3-5x}\right)\right]$$

$$\underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{1}{2}\frac{d}{dx}\left[\ln\left(\frac{12x^4-4x^2+2}{6x^3-5x}\right)\right] \underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{1}{2}\frac{d}{dx}\left[\ln\left(12x^4-4x^2+2\right)-\ln\left(6x^3-5x\right)\right]$$

$$\underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{1}{2}\left(\frac{1}{12x^4-4x^2+2}\left(48x^3-8x\right)-\frac{1}{6x^3-5x}\left(18x^2-5\right)\right) \underset{\text{rewrite}}{\nwarrow = \nearrow} \frac{24x^3-4x}{12x^4-4x^2+2} - \frac{18x^2-5}{12x^3-10x}$$

$$\boxed{\frac{d}{dx}\left[\ln\left(\sqrt{\frac{12x^4-4x^2+2}{6x^3-5x}}\right)\right] = \frac{24x^3-4x}{12x^4-4x^2+2} - \frac{18x^2-5}{12x^3-10x}}$$